Current-line solution for understanding and predicting B_1 / B_1^+ behavior and investigating central focusing

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Target audience. RF Engineers, Electromagnetic community in MR

Purpose. The spatial distribution of B_1 / B_1^+ field is related to the electrical and geometrical properties of the sample, the frequency of operation f_0 (and, hence, B_0) and the coil itself. The complex interactions between coils and samples cannot be usually solved using analytical methods; however, customised analytical methods can be developed using simplified geometries and assumptions. Here we propose a novel analytical approach based on the theory of cylindrical waves radiated by a filament of a-c current (dynamic solution) and illuminating a cylindrical load. We will provide the solution of the 2-D boundary value problem for an ideal shielded birdcage/ TEM resonator. The approach permits to have a physical insight into spatial distribution of B_1, B_1^+ with respect to electrical properties of the sample and frequency. Moreover, it permits to separate the solution to single line source problem (primordial solution) and the composite one (the summations of primordial solutions accordingly to the resonator driving). The capability of separating the primordial solution and the composite one is fundamental for a thorough analysis of central focusing, investigating and discriminating among phenomena of dielectric resonance, standing wave and multi-source interference.

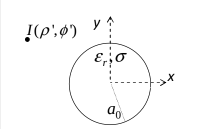

Fig 1. Homogeneous cylinder illuminated by a line source

Methods. We consider an electric current line source I parallel to z-axis and positioned, using cylindrical coordinates, at (ρ', ϕ'). We can obtain the E and H fields by letting the vector potential $\vec{A} = I/(4j) H_0^{(2)}(k_0|\vec{\rho} - \vec{\rho}'|)\hat{i}_z$, where $H_0^{(2)}$ is the Hankel function of second kind and zero order. The line source illuminates a homogeneous dielectric cylinder in free space, having radius $a_0 < \rho'$ (Fig 1a). The cylinder is characterized by relative permittivity ε_r and conductivity σ. E and H fields can be determined in each region after writing the field radiated by the line source using the addition theorem for Bessel function and after imposing the boundary conditions [1]. Turning now to an ideal shielded birdcage/TEM resonator with M rods, we assume that the rods can be represented by M electric current line sources. Image theorem is used to remove the shield. The solution for the entire resonator is built up by azimuthally displacing the primordial solution, weighting according to current distribution (which depends on the driving), and adding. The procedure permits to calculate E and H, and hence B_1. B_1^+ is derived through projection in the rotating frame.

Results and Discussions. Fig 2a and Fig2c show $|B_1|$ obtained when using a single line source excitation at 298 MHz (I=0.001A) positioned at $(\rho' = 29.5\text{cm}, \phi' = 0)$ illuminating a pure water (ε_r=80, σ=0 S/m) and a saline water (ε_r=80, σ=1 S/m) cylinder, respectively (radius = 6 cm). $|B_1|$ in Fig 2a shows a behavior clearly related to standing wave effects. This condition is quite far from a dielectric resonance, since dielectric resonance in a cylinder shows a maximum or a minimum in the center [1]. In Fig 2c the standing wave effect is damped by the losses. Fig 2e represents $|B_1^+|$ in the saline water cylinder: curling effect due to conductivity appears. Similarly, $|B_1|$ is computed for the an ideal shielded birdcage/TEM resonator having a quadrature driving at 298 MHz, fundamental mode, with M=16 rods displaced along a circle of radius of 29.5 cm (equally spaced along the azimuth) and with the radius of the shield of 37.5 cm. Fig 2b refers to pure water cylinder while Fig 2d shows the saline cylinder results. Both Fig 2b and Fig 2d show a focusing in the central region due to multi source interference; focusing in Fig 2d is damped by the conductivity. Fig 2f represents $|B_1^+|$ for saline water cylinder; note that the curling effect due to conductivity disappeared because of the exact quadrature driving.

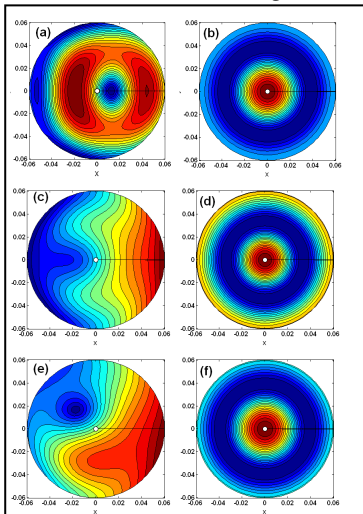
Fig 2. $|B_1|$ and $|B_1^+|$ at 298 MHz when using a single line source (left column) and quadrature (right column) excitation: a-b) $|B_1|$ inside a pure water cylinder; c-d) $|B_1|$ inside a saline water cylinder; e-f) $|B_1^+|$ inside a saline water cylinder

Dielectric resonance appears for particular values of frequency, geometric relationships between coil and load, and dielectric properties of the load. If a homogeneous cylinder is considered as load, resonance frequencies are expected at the zeros of Bessel functions (this holds true either without the driving term [2] or when ε_r >>1 [1]). Fig 3a shows $|B_1|$ obtained when using a single line source excitation at 214 MHz positioned at $(\rho' = 29.5\text{cm}, \phi' = 0)$ illuminating the pure water cylinder, while in Fig 3c a frequency of 341 MHz is employed. 214 MHz and 341 MHz correspond to the first and the second dielectric resonance frequencies. $|B_1|$ obtained using a single line source excitation at 298 MHz has been already shown in Fig 2a. Fig 3a and 3c demonstrate that when a resonance frequency is used, the pattern of the correspondent mode is visible in magnitude, as opposed to Fig 2a that represents the magnitude for an operating frequency of 298 MHz, which is not a dielectric resonance frequency. In Fig 2a the magnitude does not permit to distinguish any modes; the behavior of B_1 is clearly related to standing wave effects. Fig 3b, Fig 2b and Fig 3d show $|B_1|$ calculated for quadrature driving at 214 MHz, 298 MHz, 341 MHz; in all these figures it is possible to note a focusing effect in the center, which leads to a focusing in $|B_1|$ and, hence, to the phenomenon of central brightening. Note that focusing in the composite solution is due to multi-source interference and appears with and without dielectric resonance; the same effect has been observed in [3]. In [3] the analysis has been performed using circular surface coils; conversely, here we identified the primordial solution, i.e. the solution to single line source.

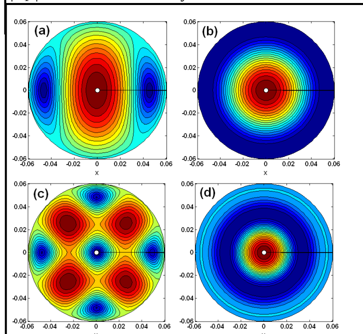
Fig 3. $|B_1|$ inside a pure water cylinder using a single line source (left column) and quadrature (right column) excitation: a-b) first dielectric resonance (214 MHz); c-d) second dielectric resonance (341 MHz)

Finally, one further simulation which permits to clearly identify dielectric resonance has been performed. Specifically, Fig 4 shows $|B_1|$ obtained when using a single line source excitation at 214 MHz, 298 MHz and 341 MHz positioned closer to the cylinder, i.e. at $(\rho' = 9.5\text{cm}, \phi' = 0)$. By comparing Fig 4a with Fig 3a and Fig 4c with Fig 3c it is possible to observe the same $|B_1|$ pattern. It follows that at 214MHz and 341 MHz, a dielectric resonance appears and the pattern of the correspondent mode is visible in B_1 magnitude regardless the distance of the line source excitation. This does not hold at 298MHz, as argued by comparing Fig 4b with Fig 2a. It follows that at frequencies that are far away from dielectric resonance frequencies, as it is the case for 298 MHz, the magnitude maps are dominated by standing wave effects.

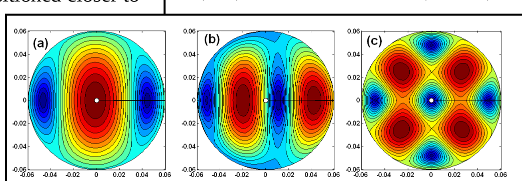
Fig 4. $|B_1|$ inside a pure water cylinder using a single line source excitation positioned very close to the cylinder: a) 214 MHz; b) 298 MHz; c) 341 MHz

Conclusion. The approach permits the separation of the single line solution and the composite one and allows the characterization of dielectric resonance, standing wave, multi-source interference. Focusing can be caused by multi-source interference and is present both with and without dielectric resonance.

References. 1. Harrington, R. F. , *Time-Harmonic Electromagnetic Fields*, McGraw-Hill Book Company, 1961, 2. J. Tropp, *J Magn Reson* 167,12-24, 2004, 3. Collins C. M., Liu W., Schreiber W., Yang Q. X. and Smith M. B., *J Magn Reson Imag* 21-2, 192-196, 2005.