Simple, accurate and efficient multilayer integral method for eddy current simulation in thin volumes of arbitrary geometry produced by MRI gradient coils

Hector Sanchez Lopez[1], Fabio Freschi[2], Adnan Trakic[1], Elliot Smith[1], Jeremy Herbert[1], Miguel Fuentes[1], Stephen Wilson[1], Limei Liu[1], Maurizio Repetto[2], and Stuart Crozier[1]

[1]*ITEE, The University of Queensland, Brisbane, QLD, Australia,* [2]*Department of Energy, Politecnico di Torino, Torino, Torino, Italy*

Introduction: Eddy currents are invoked when the time-varying magnetic fields produced by the gradient coils interact with the surrounding conducting structures of the MRI scanner including RF coils. These currents in turn produces acoustic noise, power heating, magnetic field asymmetries, unpleasant acoustic noise, electronic malfunctioning, frequency shift in the RF coil and imaging artifacts [1]. In this abstract we present a new fast, accurate and efficient eddy current simulation method capable of calculating induced currents in thin (finite thickness) conducting and non-magnetic volumes of arbitrary geometry induced by arbitrary arrangements of gradient coils. We assumed that one of the linear Cartesian dimensions is much smaller than the rest and that the volume is divided in thin layers along the smallest Cartesian dimension. This novel method has been experimentally validated using a z-gradient coil and its performance tested against COMSOL and the Fourier Network method (FNM) [2]. We present an example to demonstrate the capabilities of the method in terms of predicting the induced currents, power losses and pre-emphasis simulations using the excited eigenvalue corresponding to the surrounding structure. The method is accurate and fast enough to be performed in a laptop (Intel core(TM) i7 CPU) 8 GB RAM.

Method: We assumed a thin but finite thickness, smooth, non-magnetic and conducting domain of arbitrary shape is immerse in a time-varying magnetic field produced by a known current source $\mathbf{J}_s(\mathbf{r},t)$ of arbitrary geometry. The displacement current is much smaller than the conduction current at the given frequency $\omega=2\pi f$ where f is given in Hz. The domain is divided along the smallest dimension into N layers of thickness h, where h is much smaller than the skin depth δ [2]. Each surface is approximated to a connected set of discrete mesh of plane triangles and the surface current density $\mathbf{J}_i^s(\mathbf{r},t)$ is represented as a finite set of linear basis functions [3]. The Stokes theorem holds in each thin layer and no current flows through the boundaries containing each layer; hence that the layers are inductively coupled but resistively decoupled. The boundary conditions and the edges of the domain are enforced to satisfy the continuity equation. The differential form of the diffusion equation is solved for time-harmonic or transient solution when the coil is driven with an arbitrary current pulse s(t):

$\frac{d}{dt}\left(e^{\lambda t}\mathbf{U}^{-1}\mathbf{v}_i(t)\right) = -e^{\lambda t}\mathbf{u}\frac{\partial s(t)}{\partial t}$ where $\mathbf{u} = \mathbf{U}^{-1}\mathbf{M}_{ii}^{-1}\mathbf{M}_{is}$ is a vector containing the subset of eigenmodes $\mathbf{U}$ excited by the coil with current density $\mathbf{J}_s(\mathbf{r},t)$ and inductive coupling $\mathbf{M}_{is}$. $\mathbf{v}_i(t)$ is a vector containing the unknown stream function amplitude and $\mathbf{M}_{ii}$ is the self-inductive coupling. An analytical-numerical approach is used to ameliorate the singularities contained in the inductance calculation.

A 34-turn unshielded z-gradient coil with a radius of 125.5mm, wire diameter of 1mm was designed and built. The z-gradient coil was excited with a sinusoidal current of amplitude 1.48A at 1kHz. Joule power loss, computing time and condition number were first calculated in order to demonstrate the solution stability. The conducting cylinder was divided in 5 layers and was discretized using 27,000 triangles. A trapezoidal pulse of amplitude 1.48 A (200µs rise time, 20.2ms pulse duration) was applied to induce eddy currents in a 2.5mm thick cylindrical conductor with an inner radius of 175mm, overall length of 387mm and electrical conductivity of 32.26MSm^{-1}. A low-noise TMR STJ-220 magnetic field sensor with AL-05 signal conditioning (MicroMagnetics) measured the overall magnetic field (B_z + $B_{z\ eddy}$) while the National Instruments PCI-6221 data acquisition card recorded the measurements in LabVIEW (NI) at 250kSs^{-1}. The flat-top time duration (20.2 ms) was chosen to be long enough to guarantee that no field produced by the eddy currents was present at the end of the excitation, thus B_z generated by the coil was measured. $B_{z\ eddy}$ can be obtained by subtracting the primary field $B_{z\ coil}$ from the measured total field $B_{z\ tot}$ (i.e. $B_{z\ eddy} = B_{z\ tot} - B_{z\ coil}$). The total field was measured at multiple positions (i.e. z = 0-60mm) in increments of Δz = 5mm along the +z-axis. For this purpose a single computer numerically controlled (CNC) axis (ISEL AG, Germany) was used to accurately translate the probe while the field measurements were performed. The field amplitude and the time decay constant corresponding to each axial position were obtained by using a single exponential fitting. The skin effect was calculated and compared with the FNM and COMSOL, but this time a single loop was used with current 2 A with f=100Hz,1kHz and 10kHz. The Matlab functions *bilinear* and *filter* were employed to obtain the filter characteristics, while *eig* was used to calculate the excited eigenmodes and time decay constant $\mathbf{u}$ and $\boldsymbol{\lambda}$, respectively.

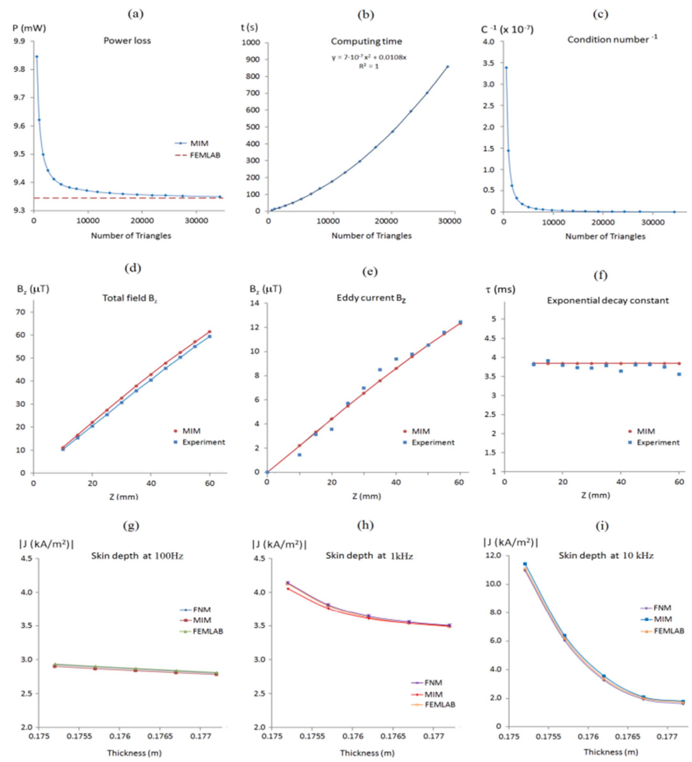

Fig 1.(a-c) Numerical stability, (d-f) validation of the MIM and (g-i) skin effect.

Results and Discussions: (Fig.1 (d-f)), are characteristic curves that signify the numerical stability of the proposed multilayer integral method (MIM). As the number of triangles is increased, the power loss values converge to a stable solution; the condition number increases to infinity and the computing time grows quadratically. This behaviour is representative of a stable numerical system for electromagnetic analysis. Fig 1 (d-f) shows the accuracy of MIM on predicting the field produced by the z-coil and the field produced by the induced currents at each axial position. Fig1 (f) depicts the behaviour of the time decay constant which is constant at each axial position, as it was expected. The MIM is capable of calculating the skin effect for very low frequency up to 10 kHz; which indicates that the new MIM is an accurate method to evaluate the eddy current diffusion process in complex structures such as an MRI scanner cryostat. Fig 2 (a) shows the only eigenmode excited by the z-gradient coil. The time decay constant was 3.805 ms which corresponds closely to the predicted and measured (Fig. 1 (f)) time decay constant. Based on the eigenvalue information ($\boldsymbol{\lambda}$) obtained by the convolution $e^{\lambda t}\mathbf{u}$ it is possible to accurately design a filter to tailor the current pulse. Fig.2 (b) describes the total field produced by the coil and the eddies; after applying a filter with time constant of 3.85 ms with a current overshot of 23 % the resulting residual $B_{z\ eddy}$ was reduced about 50 times.

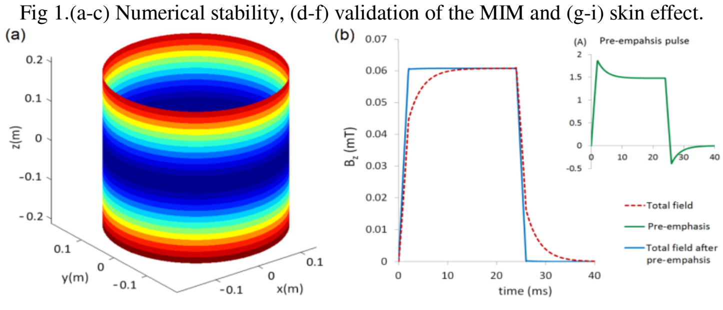

Fig 2.(a-b) Eigenmode excited by the z-gradient coil and current pre-emphasis.

Conclusions: An efficient and accurate multilayer integral method to simulate eddy currents induced by coils of arbitrary geometry in thin but finite thickness conducting structures of arbitrary geometry has been presented. The method was validated experimentally.

References: [1] C. S. Levin and H. Zaidi, *PET Clin,* vol. 2, pp. 125-160, 2007. [2] H Sanchez-Lopez *et al. Journal of Magnetic Resonance*, 207, 251-261, (2010). [3] S. Pissanetzky, *Meas. Sci. Technol.,* vol. 3, pp. 667-673, 1992. [4] H Sanchez-Lopez *et al Journal of Magnetic Resonance*, 199, 48-55, (2009).

Acknowledgements: This work was performed for MedTeQ, a Queensland Smart State funded research centre.