Fourier Domain Approximation for Bloch Siegert Shift

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Target audience : Researchers using the B_1 mapping method based on Bloch-Siegert shift.

Purpose : In this study, we propose a new simple Fourier domain based analytical expression for the Bloch-Siegert(BS) phase shift based B_1 mapping method. In this expression, the phase is calculated in terms of Fourier transform of the RF pulse envelope and therefore off- and on-resonance effects can be understood more easily. It is shown that $|B_1^+|$ can be obtained more accurately by the aid of this expression while employing short BS pulse durations and small off-resonance frequencies.

Theory: In the BS shift based B_1 mapping method, off-resonant RF pulse is applied after the excitation RF pulse in order to add phase shift to the excited spins. The amount of the phase shift depends on the envelope of the applied RF field (B_1^+ (t)) and the frequency offset of the RF pulse (ω_{RF} (t)) from the resonance frequency (ω_0). In [1], it is shown that when $\omega_{RF}(t)$ is much higher than $|\omega_1(t)| = \gamma|B_1^+(t)|$ where γ is the gyromagnetic ratio, then in the ω_0 rotating frame, the phase shift is directly related to the time integral of the square of $|\omega_1(t)|$ and inversely related to the offset frequency as given in Eq. (1).

$$\phi_{TD} = \int_0^T \frac{|\omega_1(t)|^2 dt}{2\omega_{RF}(t)}\text{d}t \approx \int_0^T \omega_{BS}(t)\text{d}t \quad ..(1)$$

$$\phi_{res} = \phi_{actual} - \int_0^T \omega_{BS}(t)\text{d}t \quad ....(2)$$

Since long BS pulse durations cause long TE values and since longer TE values result in signal loss due to the T_2* effects, use of small pulse duration becomes important. In an earlier study [2], it was shown that when small pulse duration is used, there is a significant difference between the actual phase shift (ϕ_{actual}), and the time-domain approximation given by Eq. 1. This residue (ϕ_{res}) is defined as in Eq. (2). ϕ_{res} can be calculated if the Bloch equations are solved in the ω_0+ω_{BS}(t) rotating frame, since by doing so the phase accumulating due to ω_{BS}(t) is excluded from the actual phase shift in the ω_0 rotating frame. In order to simplify the solution of the Bloch equations, a new magnetization vector which includes the phase changes until the beginning of the Bloch-Siegert pulse is defined with the initial condition $(M_x(0)\ M_y(0)\ M_z(0))=(M_0\ 0\ 0)^T$. In this condition the time derivative of M_x is very small, and it is assumed that M_x remains almost constant throughout the Bloch-Siegert RF pulse. Therefore the system of differential equations is reduced to Eq.(3) where $\omega_{1x}(t)$ and $\omega_{1y}(t)$ are the real and imaginary parts of $\omega_1(t)$, respectively. By using the $\omega_{RF}(t) >>|\omega_1(t)|$ approximation the solution for the M_y component for the pulse duration T can be written as in Eq. (4). Since we assume that $M_x = M_0$ and M_y is small, ϕ_{res} becomes approximately equal to $-M_y/M_0$, when ϕ is defined in left-hand direction. In order to find the phase shift defined in the ω_0 rotating frame (ϕ_{actual}), we can add $\int\omega_{BS}(t)dt$ term to ϕ_{res} as given in Eq. 2. However, due to $\omega_{RF}(t) >>|\omega_1(t)|$ approximation, the final phase expression given in Eq. 5 is again the approximated solution for ϕ_{actual} and defined as the frequency domain BS approximated phase shift ϕ_{FD}. In this expression $\Omega_1(f)$ is the Fourier transform of $\omega_1(t)$ and since the Hilbert transform of a function g(t) at t = 0 is given as $-1/\pi \int g(\tau)/\tau d\tau$, the final form of the expression is defined in terms of the Hilbert transform. In order to find the peak of B_1 field from the phase in $\omega_{RF}(t) >>|\omega_1(t)|$ region Eq. 5 is changed to Eq.6 where $\Omega_1(f)= \gamma\, B_{1peak}\, \Omega_{norm}(f)$.

$$\frac{d}{dt}\begin{pmatrix} M_y \\ M_z \end{pmatrix} = \begin{pmatrix} 0 & \omega_{1x}(t) \\ -\omega_{1x}(t) & 0 \end{pmatrix}\begin{pmatrix} M_y \\ M_z \end{pmatrix} + \begin{pmatrix} \omega_{BS}(t) \\ \omega_{1y}(t) \end{pmatrix} M_0 \quad ..(3)$$

$$M_y \approx M_0 \int_0^T \omega_{BS}(t)dt + M_0 \int_{-T/2}^{T/2}\int_t^{T/2} \omega_{1y}(t)\omega_{1x}(s)\text{d}s\text{d}t \quad (4)$$

$$\phi_{actual} \approx \phi_{FD} = -\int_{-\infty}^{\infty}\left[\frac{|\Omega_1(f)|^2}{4\pi f}\right]df = \frac{H|\Omega_1|^2(0)}{4} \quad (5)$$

$$B_{1peak} \approx \frac{1}{\gamma}\sqrt{\frac{4\phi_{FD}}{H|\Omega_{norm}|^2(0)}} \quad ....(6)$$

Experiments & Results: In order to verify the frequency domain BS approximation (Eq. 5), Bloch simulations and MR experiments are performed for Hard, Fermi and Shinner-Le Roux (SLR) pulse shapes with different pulse durations. SLR pulse is designed with 0.5% passband ripple, 1% reject ripple, and 0.3 kHz bandwidth by using VESPA-RFPulse tool [3]. All experiments were performed using a 3 tesla Siemens Tim Trio scanner with a Siemens phantom. The imaging parameters were: slice thickness=5mm, FOV=200mm, TR=100ms and resolution=256x256. The body coil was used for RF transmission and a 12-channel Siemens head coil was used for the reception. For hard and Fermi pulse shapes, the pulse duration was varied between 150µs and 2ms, and for SLR pulse shape, the duration was varied between 300µs and 2ms. For each pulse shape the offset frequency is fixed at 4 kHz. In Figure 1, we present a comparison of the phase shifts obtained through Bloch simulations, observed in the experiments, obtained by Eq. 1, and obtained by Eq. 5 for different pulse durations. As seen in the figure, the results of the experiments follow the results of the Bloch simulations as expected and the difference between the results of frequency domain approximation (Eq. 5) and the simulations is negligible. However, there is an appreciable phase error between the results of the time domain expression (Eq. 1) and the simulations.

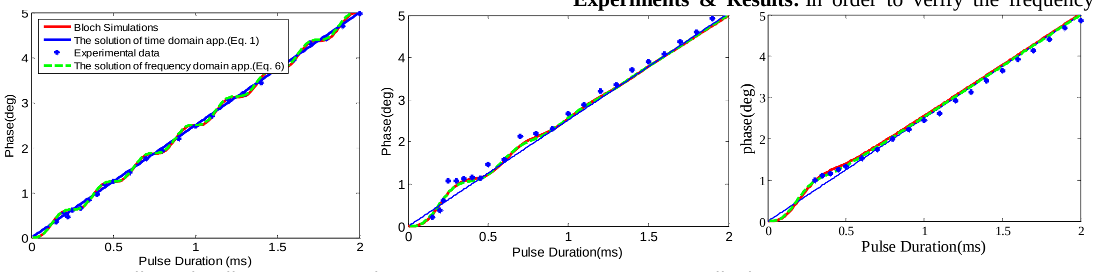

Figure 1: Phase difference for different pulse durations for Hard pulse, Fermi pulse, SLR pulse with *4kHz* offset frequency.

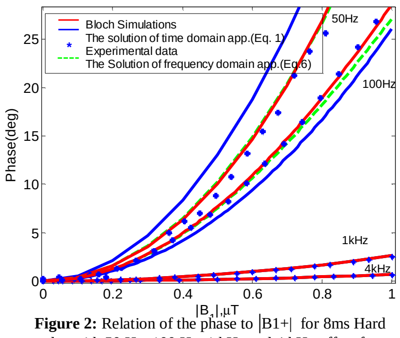

Figure 2: Relation of the phase to |B1+| for 8ms Hard pulse with 50 Hz, 100 Hz, 1 kHz and 4 kHz offset freq.

%	Error analysis with Simulations		Error analysis with Experiments	
	ϕ_{TD}	ϕ_{FD}	ϕ_{TD}	ϕ_{FD}
For 4kHz	0.4%	0.4%	4%	4%
For 1kHz	0.1%	0.05%	7%	7%
For 100Hz	15%	3.25%	14%	3.5%
For 50Hz	25%	2.5%	30%	9%

Table 1: Error Analysis of the solutions of ϕ_{TD} & ϕ_{FD} relations with the results of simulations &experiments

To analyze the relation between the phase and $|B_1^+|$ at different offset frequencies, experiments were performed by using hard pulse with offset frequencies of 50 Hz, 100 Hz, 1 kHz and 4 kHz with $|B_1^+|$ values in $\omega_{RF}(t) >>|\omega_1(t)|$ region. The phase shift values that were obtained using the frequency domain (Eq. 5) and the time domain (Eq. 1) approximations were compared with the results of simulations and experiments (see Figure 2). Although at 1 and 4 kHz frequencies, all results match very closely, when the offset frequency is 100Hz, the results of Eq. 1 start to deviate from the results of Bloch equations and the experimental results whereas Eq. 5 gives similar results with the experiments. Table 1 shows the error analyses done by using $\phi_{error} = \sum_{n=1}^{N} \frac{\phi_{1n} - \phi_{2n}}{\phi_{1n}} \times \frac{100}{N}$ relation for N=20 $|B_1|$ data points for each frequency. These error analyses also verify that the frequency domain approximated relation gives better solution for $|B_1^+|$ than the solution of the time domain approximated relation for lower offset frequencies. It should be noted that at low BS frequencies, precise knowledge of the B_0 field and therefore the frequency offset is critically important. In addition, use of crusher gradients is very important to reduce image artifacts.

Conclusion: In this study, a new simple frequency domain analytical expression is proposed for the BS shift. Using this expression, $|B_1^+|$ values can be predicted from the phase data by using the frequency spectrum of the RF pulse. The method works well even for short pulse durations and offset frequencies.

References: [1] Sacolick et al. MRM 2010,63:1315-1322. [2] Turk et al. Proc ISMRM 20:608 (2012). [3] Matson et al. MRM 1994,12: 1205-1225.