Large tip angle k_T-points based on a linearization of the Bloch equations

Florent Eggenschwiler[1], Rolf Gruetter[2], and José P. Marques[3]
[1]*EPFL, Laboratory for Functional and Metabolic Imaging, Lausanne, Vaud, Switzerland,* [2]*Universities of Geneva and Lausanne, Department of Radiology, Lausanne, Vaud, Switzerland,* [3]*University of Lausanne, Department of Radiology, Lausanne, Vaud, Switzerland*

Introduction: K_T-points [1] is a promising technique for correcting the inhomogeneous B_1^+ distribution resulting from the short wavelength present at high field strengths ($B_0 \geq 3T$). The k_T-points design was first proposed in the small tip-angle (STA) regime and was then extended to large tip-angles using the optimal control approach [2] which is highly demanding in terms of computation resources. A faster methodology would be highly desirable, especially when combining k_T-points with turbo spin echo (TSE) sequences with variable flip angles [3] for which several high tip-angle k_T-points have to be designed. This work presents a new approach for designing high tip-angle k_T-point pulses based on a linearization of the Bloch equations and usage of symbolic notation to accelerate the computation when the optimization has to be performed for large number of pixels.

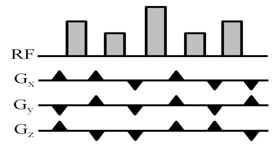

Fig. 1: Set of symmetric sub-pulses and gradients (N=5 k_T-points).

Methods: A k_T-point trajectory consists of a set of N RF sub-pulses (SP) and 3(N+1) gradient waveforms defined by the k_x, k_y and k_z positions of the k_T-points (Fig.1). Considering the matrix formalism of the Bloch equations [4], the states of the magnetization before and after a k_T-point pulse are linked by the system of equations presented in Fig. 2. The Cayley-Klein parameters α and β are given by: $\begin{pmatrix}\alpha\\ \beta\end{pmatrix} = Q^{Tot}\cdot\begin{pmatrix}1\\ 0\end{pmatrix}$, with $Q^{Tot}=Q^{G,N+1}\cdot Q^{RF,N}\cdot\ldots\cdot Q^{G,2}\cdot Q^{RF,1}\cdot Q^{G,1}$, describing the succession of RF sub-pulses and gradient waveforms applied during the k_T-point pulse. Each Q matrix has the form: $\begin{pmatrix}a & b\\ -b^* & a^*\end{pmatrix}$, with $a=\cos(\varphi/2)-i\cdot n_z\sin(\varphi/2)$ and $b=-i(n_x-i\cdot n_y)\sin(\varphi/2)$, describing a rotation through an angle φ about a vector $\mathbf{n}=(n_x, n_y, n_z)$. In this specific problem, α and β coefficients are functions of the amplitudes and phase of the k_T-point weights as well as their k_x, k_y, k_z positions. The optimization of these parameters was done by using a linear extension of the targeted coefficients α^{aim} and β^{aim}: $\alpha^{aim}=\alpha+\partial\alpha/\partial P\cdot\Delta P$ and $\beta^{aim}=\beta+\partial\beta/\partial P\cdot\Delta P$, where $\partial()/\partial P$ is the partial derivative in respect to either the real and imaginary part of each SP or k_x, k_y, k_z steps between SPs. ΔP corresponds to the increment of those parameters. $\partial\alpha/\partial P$ and $\partial\beta/\partial P$ can be calculated using: $\partial Q^{Tot}/\partial P_i=Q^{G,N+1}\cdot\ldots\cdot\partial Q^{RFi}/\partial P_i\cdot\ldots\cdot Q^{RF,1}\cdot Q^{G,1}$ for the SP parameters and $\partial Q^{Tot}/\partial P_i=Q^{G,N+1}\cdot\ldots\cdot\partial Q^{Gi}/\partial P_i\cdot\ldots\cdot Q^{RF,1}\cdot Q^{G,1}$ for k_x, k_y and k_z.

$$\begin{bmatrix}M_{xy}\\ M_{xy}^*\\ M_z\end{bmatrix}^+ = \begin{bmatrix}(\alpha^*)^2 & (\beta^*)^2 & -2\alpha^*\beta^*\\ -\beta^2 & \alpha^2 & -2\alpha\beta\\ \alpha^*\beta & \alpha\beta^* & \alpha\alpha^*-\beta\beta^*\end{bmatrix}\cdot\begin{bmatrix}M_{xy}\\ M_{xy}^*\\ M_z\end{bmatrix}^-$$

Fig. 2: Bloch equations in the spin domain written with the matrix formalism proposed in [4].

For any specific number of sub-pulses α, β, $\partial\alpha/\partial P$ and $\partial\beta/\partial P$ can be evaluated symbolically using MATLAB, stored as functions and then calculated for all pixels without the need of expensive loops over all pixels with small matrix operations.

Excitation optimization: Given that the initial magnetizatoin is $M_z = 1$, optimizing a 90° excitation pulse consists of making the "$-2\alpha^*\beta^*$" term of the matrix (Fig. 2) homogeneous across the brain. The difference between the target magnetization $M_{xy,T}$ ($= -2\alpha^{aim,*}\beta^{aim,*}$) and a current solution ($M_{xy,A}$) can thus be written as:

$M_{xy,T}-M_{xy,A} = -2(\alpha^*+\partial\alpha^*/\partial P\cdot\Delta P)(\beta^*+\partial\beta^*/\partial P\cdot\Delta P)+2\alpha^*\beta^* \approx -2(\alpha^*\partial\beta^*/\partial P+\beta^*\partial\alpha^*/\partial P)\Delta P$

corresponding to a linear system of equations (two equations per pixel in order to separate real and imaginary components) for which a solution ΔP can be found using the pseudo-inverse method. A full Bloch simulation of the M_{xy} profile (evaluation of $-2\alpha^*\beta^*$) is then performed with the new solution, defining an updated $M_{xy,A}$ map closer to the target $M_{xy,T}$. The procedure is repeated until a stopping criterion is reached.

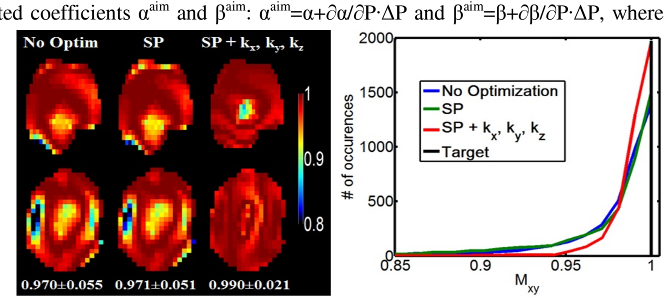

Fig. 3: Profiles and histograms of the transverse magnetization (M_{xy}) at the end of the 5 k_T-points 90° excitation pulse before and after optimization of the SP only and both SP and k_x, k_y, k_z parameters. Mean ± std across the brain are presented below each profile.

Other Optimizations: the design of inversion or refocusing k_T-point pulses seeks the homogenization of $\alpha\alpha^*-\beta\beta^*$ (transfer M_z into $-M_z$) or $-\beta^2$ (transfer M_{xy} into M_{xy}^*) respectively.

Proof of principle: A 5 k_T-points 90° excitation pulse was designed with the proposed method by optimizing first k_T-points weights (SP parameters) only and then both SP and positions k_x, k_y, k_z. A 180° refocusing pulse with 7 k_T-points was similarly designed. When designing the pulses, the iterative procedure was stopped if the $M_{xy,A}$ at iteration k+1 is not closer to $M_{xy,T}$ than $M_{xy,A}$ at step k (local minimum found) and a maximum number of 50 iterations was considered.

I*n-vivo* B_1^+ maps acquired using the SA2RAGE sequence [5] on a 7T Siemens system were used. A first estimation of the N k_T-points positions and weightings was found using the SOLO algorithm [6]. It was observed that exciting the k-space trajectory defined by the optimized k_T-points in a symmetric way: the **k=0** position forced at the center of the trajectory and the remaining k_T-points symmetrically split around it with their amplitude halved (Fig.1), offered a large range of validity to the STA approximation and hence a good starting point for further optimizations.

Results and discussion: Figure 3 displays the profiles and histograms of M_{xy} throughout the brain before and after optimization of SP parameters only and both SP and k_x, k_y, k_z for the 90° excitation pulse. A 63% improvement was obtained when comparing the std/mean without and with optimization of both SP and k_x, k_y, k_z parameters. Figure 4 shows the M_{xy}^* distributions before and after the design of the 7 k_T-points refocusing pulse. In this case, a 57% homogeneity improvement was reached when optimizing the SP parameters only.

Since making the k_T-point positions adjustable provides another degree of freedom, it is surprising to observe that for the refocusing pulse, optimizing the SP parameters only, provides better results than the SP + k_x, k_y, k_z optimization. It is likely that during the SP + kx, ky, kz optimization, a local minimum was reached by the iterative procedure, fulfilling the stopping criterion whereas during the SP optimization a better solution was found. The excitation and refocusing k_T-point pulses were respectively designed in 40s and 4min58s (when the best std/mean was found) on a standalone PC.

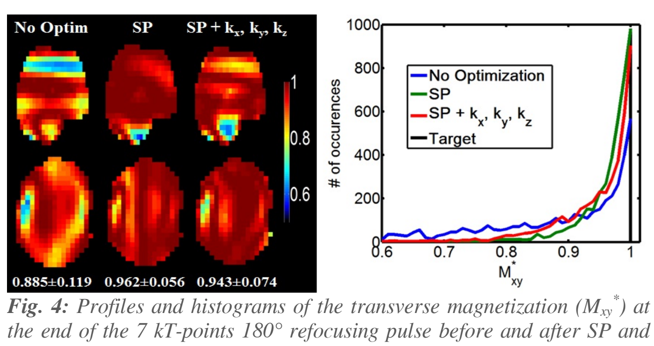

Fig. 4: Profiles and histograms of the transverse magnetization (M_{xy}^*) at the end of the 7 kT-points 180° refocusing pulse before and after SP and SP + k_x, k_y, k_z optimizations.

Conclusion: High tip-angle excitation and refocusing k_T-point pulses were designed by iteratively using a linear form of the Bloch equations (full Bloch equations for 7 k_T-points on a 32x32x24 matrix computed in 0.3secs on a standard 2.4GHz processor). Although the whole calculation of the excitation and refocusing pulse parameters was performed in less than 5mins, the procedure could be made faster by reducing the matrix size (randomly distributed subset of pixels inside the brain) or having a less stringent convergence criterion. The presented methodology could be easily extended to parallel transmission, which would provide another degree of freedom to the optimization process and be adapted to have an SAR regularization term.

References: [1] Cloos et al, MRM 67, 2012, [2] Cloos et al, ISMRM, 2012, p.634, 1955, [3] Eggenschwiler et al, ESMRMB Meeting, 2012, p.48, [4] Jaynes, Phys Rev 98, [5] Eggenschwiler et al, MRM 67, 2012, [6] Ma et al, MRM 65, 2011.

Acknowledgements: Supported by the CIBM of the UNIL, UNIGE, HUG, CHUV, EPFL and the Leenaards and Jeantet Foundations.