Spin Echo Formation with a Phase Pre-Winding Pulse

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Introduction: Standard spin echo sequences require at least one refocusing pulse (usually a π-pulse), which destroys the longitudinal magnetization. Therefore, the necessary long repetition times make T_2-weighted 3D-encoding impractical. In this work a class of global radio frequency (RF) pulses is proposed that have an inverse phase slope (compared to a standard linear phase pulse) over a finite range of Larmor frequencies. This leads to a spin echo formation after a single excitation pulse. Choosing small tip angles, the longitudinal magnetization is mostly maintained, potentially allowing T_2-weighted imaging with short repetition times.

Pulse design: Shinnar et al. proposed using finite impulse response (FIR) filters for the design of Shinnar-Le Roux (SLR) pulses [1]. In general filters have a positive group delay, which is defined as the negative gradient of the phase of the frequency response. A positive group delay corresponds to a temporal delay of the signal. FIR filters with a negative group delay over a finite frequency range have been proposed in order to predict the signal within the particular frequency range [2, 3]. Such predictive FIR filters can be used to design RF-pulses that pre-wind the phase of the spins in a way similar to the combination of excitation- and refocusing-pulse in a standard spin echo experiment. The frequency response $\vec{d}$ of a FIR filter is given in matrix form by

$$\vec{d} = \mathbf{E} \cdot \vec{x} \,. \qquad (1)$$

The forward operation is denoted by a matrix with the entries $\mathbf{E}_{k,n} = \exp(i \cdot \omega_k \cdot n \cdot \Delta t)$, where Δt is the dwell time, ω the Larmor frequency and $\vec{x}$ contains the filter coefficients. For computational simplicity an ℓ_2-norm approximation of the desired frequency response $d_k = |M_\perp| \exp(i \cdot \omega_k \cdot TE)$ is chosen. Here $M_\perp$ denotes the transversal magnetization and TE the echo time. The filter is then given by

$$\arg\min_{\vec{x}} \left\| \vec{d} - \mathbf{E} \cdot \vec{x} \right\|_2^2 + \lambda^2 \left\| \vec{x} \right\|_2^2 \,. \qquad (2)$$

Tikhonov regularization with the regularization parameter λ is applied since the problem is ill-conditioned. The solution to this minimization problem is the Moore-Penrose pseudoinverse $\vec{x} = \mathbf{E}^H (\mathbf{E}\mathbf{E}^H + \lambda^2 \mathbf{1})^{-1} \vec{d}$, where H denotes the transposed, complex-conjugate and $\mathbf{1}$ the identity. The RF-pulse can then either be calculated by the inverse SLR transform [4] or the resulting filter can directly be used as the pulse in small tip angle approximation.

Methods: Filters were designed to have a magnitude of $|M_\perp| = 0.1$ (normalized to the equilibrium magnetization $M_0 = 1$) and an echo time (measured from the end of the RF-pulse) of 1.5 ms in the range of $-1000\ rad/s \le \omega \le 1000\ rad/s$. The length of the pulse was set to 4 ms and the dwell time to 1 µs. The resulting filter was used as the RF-pulse in small tip angle approximation. The frequency response was calculated with the SLR forward model [4].

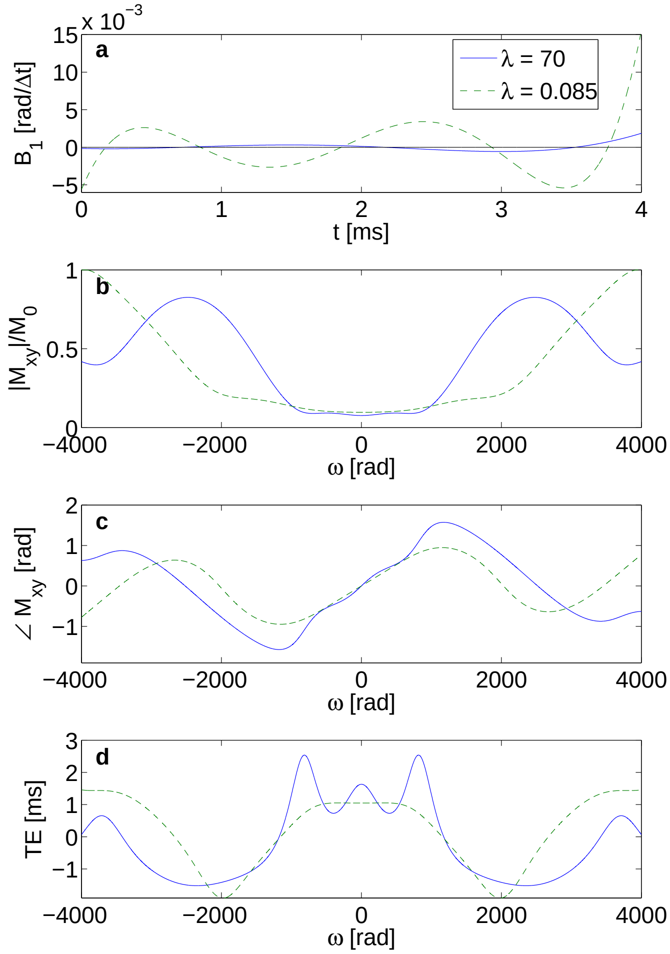

Figure: Pulses for different regularization parameters are plotted in units of flip angle per dwell time (1 µs) (**a**). The absolute value and the phase of the normalized frequency response of the pulses are shown in (**b**) and (**c**), respectively. The first derivative of the phase with respect to the frequency corresponds to the echo time (which is the negative group delay in filter theory) (**d**).

Results and Discussion: Since the desired frequency response is complex-conjugate symmetric, the resulting pulse is real valued. Tikhonov regularization minimizes the energy of the RF-pulse (Figure a), but leads to increasing ripples of the phase slope (c). Also the actual echo time is slightly lower than the desired one (d). Consistent with the prediction filter literature [2, 3], the target frequency range is a local minimum of the frequency response (b). Large tip angles outside the target frequency range lead to a suppression of the signal in steady state for short repetition times. The suppression can be further optimized by either including higher frequencies in $\vec{d}$ and introducing a weighting matrix, or employing Lagrange multipliers [3]. Future work will include exploring different pulse design methods like optimal control [5], the analysis of the contrast giving effective echo time and 3D imaging experiments with the designed pulses.

Conclusion: Global RF-pulses were proposed that lead to a spin echo formation after a single excitation pulse. Since frequencies outside the target range have large tip angles, suppression of e.g. fat is possible in steady state without an additional sequence block.

References: [1] Shinnar, M., et al., MRM 12: 81–87 (1989); [2] Heinonen, P., et al., IEEE ASSP 36; 6: 892-899 (1988); [3] Koppinen, K., et al. *Proc. IEEE IM* 1: 54-59 (1997); [4] Pauly, J., et al., *IEEE MI* 10; 1: 53-65 (1991); [5] Conolly, S., et al., *IEEE MI* 5; 2: 106-115 (1986)

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