Multidimensional Shinnar-Le Roux RF Pulse Design

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INTRODUCTION: The SLR method [1] is often known as the "method-of-choice" in 1D RF pulse design. It is desirable to generalize the SLR method to design multidimensional RF pulses. However, significant technical difficulties arise in generalizing the SLR method to multi-dimensions, largely due to the obstacle in mapping the coefficients of multidimensional polynomials uniquely to an RF pulse [2]. In this work, we present a novel approach to generalize the SLR method to multi-dimensions. The multidimensional RF pulse design problem is converted to a series of 1D RF pulse design problems, each of which is equivalent to a 1D polynomial design problem that can be solved efficiently using convex optimization.

METHODS: For simplicity, the proposed method is described for designing a 2D spatially selective RF pulse. Assume that B_1 and B_0 field is uniform, and that an "echo-planar" gradient waveform is used as shown in Fig. 1a. The pulse can be decomposed into segments in Fig. 1b. Each segment consists of two sequential rotations: a rotation varying along the x-axis caused by a G_x gradient blip, and a rotation varying along the z-axis caused by an RF subpulse along with an inherently refocused G_z gradient. For each z, the above decomposition presents an interleaved rotation synthesis structure that is the same as the 1D SLR method. The 2D RF pulse design problem can then be converted into two sequential 1D polynomial design problems: 1) design of the desired rotation of each subpulse to approximate a target excitation pattern; and 2) design of each subpulse to approximate the desired rotation determined in step 1).

To solve the first design problem, a forward and inverse transform (Table 1) are derived to establish a bijective mapping between the desired rotation of each subpulse (represented by the Caylay-Klein parameters [1]) and two 1D polynomials. Moreover, the forward transform in (1) is exact, and the rotation of each subpulse can be large. The first design problem is therefore equivalent to a 1D polynomial design problem. For each z, B_{nx} that is a polynomial of $e^{-i\omega_x}$ is designed to approximate the target excitation pattern (a function of x) in a minimax fashion. The resulting optimization problem is convex, and can be efficiently solved using a convex optimization solver, *e.g.*, CVX [3]. The polynomial A_{nx} is then designed by spectral factorization [1], and the desired rotation is finally determined using the inverse transform in (2-3). Repeating the above design along the z-axis, the desired rotation of each subpulse is obtained as a function of z, *i.e.*, $C_{nx,d}(z)$ and $S_{nx,d}(z)$.

To solve the second design problem, the rotation of each subpulse is converted to two polynomials in (4-5) using the forward SLR transform [1]. To design a minimum RF power subpulse, only the polynomial S_{nx} is designed, and C_{nx} is determined by spectral factorization in (6). However, by doing this, we lose the control of the phase of C_{nx}. That is, an arbitrary pair of $C_{nx,d}$ and $S_{nx,d}$ cannot be approximated simultaneously. Fortunately, the phase of the desired rotation $C_{nx,d}$ is a free parameter in (3), and can be chosen such that as S_{nx} approximates $S_{nx,d}$ in (8-9), C_{nx} determined in (6) also approximates $C_{nx,d}$ in (7). The polynomial S_{nx} is then designed to approximate the desired rotation $S_{nx,d}$ in (8-9) in a minimax fashion, which also results in a convex optimization problem. The subpulse can be finally determined by the inverse SLR transform [1].

Given the approximation ripples of the two sequential design problem, the overall ripples of the resulting excitation pattern can be derived based on a perturbation analysis (the exact expression is not shown due to space limitation). Explicit tradeoffs among the design parameters can be made in a similar way to the 1D SLR method [1].

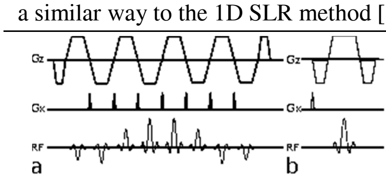

Figure 1. a: Proposed pulse **b:** Subpulse that consists of two rotations: $\begin{bmatrix} C_{nx}(z) & -S_{nx}^*(z) \\ S_{nx}(z) & C_{nx}^*(z) \end{bmatrix} \begin{bmatrix} e^{i\omega_x/2} & 0 \\ 0 & e^{-i\omega_x/2} \end{bmatrix}$.

Table 1. Equations in the first step

Forward transform:

$$\begin{bmatrix} A_{nx} \\ B_{nx} \end{bmatrix} = \begin{bmatrix} C_{nx,d}(z) & -S_{nx,d}^*(z)e^{-i\omega_x} \\ S_{nx,d}(z) & C_{nx,d}^*(z)e^{-i\omega_x} \end{bmatrix} \begin{bmatrix} A_{nx-1} \\ B_{nx-1} \end{bmatrix} \quad (1)$$

Inverse transform:

$$\begin{bmatrix} A_{nx-1} \\ B_{nx-1} \end{bmatrix} = \begin{bmatrix} C_{nx,d}(z) & -S_{nx,d}^*(z)e^{-i\omega_x} \\ S_{nx,d}(z) & C_{nx,d}^*(z)e^{-i\omega_x} \end{bmatrix}^H \begin{bmatrix} A_{nx} \\ B_{nx} \end{bmatrix} \quad (2)$$

$$S_{nx,d}(z)/C_{nx,d}(z) = A_{nx}^{(0)}(z)/B_{nx}^{(0)}(z) \quad (3)$$

Table 2. Equations in the second step

$$C_{nx}(z) = \textstyle\sum_{n_z}^{N_{z-1}} a_{n_z}^{(nx)} e^{-i\omega_z n_z} \quad (4)$$

$$S_{nx}(z) = \textstyle\sum_{n_z}^{N_{z-1}} b_{n_z}^{(nx)} e^{-i\omega_z (n_z - N_z)/2} \quad (5)$$

$$C_{nx}(z) = \sqrt{1 - |S_{nx}(z)|^2} e^{i\mathcal{H}\{\log\sqrt{1-|S_{nx}(z)|^2}\}} \quad (6)$$

$$C_{nx,d}(z) = \sqrt{1 - |S_{nx,d}(z)|^2} e^{i\mathcal{H}\{\log\sqrt{1-|S_{nx,d}(z)|^2}\}} \quad (7)$$

$$\angle S_{nx,d}(z) = \angle C_{nx,d}(z) + \angle\left(A_{nx}^{(0)}(z)/B_{nx}^{(0)}(z)\right) \quad (8)$$

$$|S_{nx,d}(z)| = \sin\left(\tan^{-1}\left|A_{nx}^{(0)}(z)/B_{nx}^{(0)}(z)\right|\right) \quad (9)$$

RESULTS: The proposed method was used to design a 2D 180° refocusing RF pulse to refocus spins in a rectangular region, and compared with the separable design method in [4], which is largely derived based on a similar intuition and essentially assumes the rotation of each subpulse is small. The simulated excitation patterns ($|\beta|^2$), are shown in Fig. 2b and Fig. 2c. While the excitation pattern achieved by the method in [4] has notable geometric distortions, the excitation pattern achieved by the proposed method approximates the desired one more accurately. Two representative excitation profile plots are shown in Fig. 2d and Fig. 2e, where the proposed method shows equal-ripple and much smaller excitation errors.

CONCLUSION: The SLR method for 1D RF pulse design has been extended to the multidimensional case. The proposed method preserves almost all the desirable features of the SLR method in terms of handling the nonlinearity of the Bloch equation and tradeoffs among design parameters and computational efficiency.

REFERENCES:[1] J. Pauly, *et al.*, IEEE TMI 1991;10:53-65. [2] W. Grissom, *et al.*, Magn Reson Med 2012;68:690-702. [3] M. Grant, *et al.*, CVX 2011. [4] J. Pauly, *et al.*, Magn Recon Med 1993;29:776-782. [5] S. Conolly, *et al.*, J Magn Reon 1988;78:440-458.

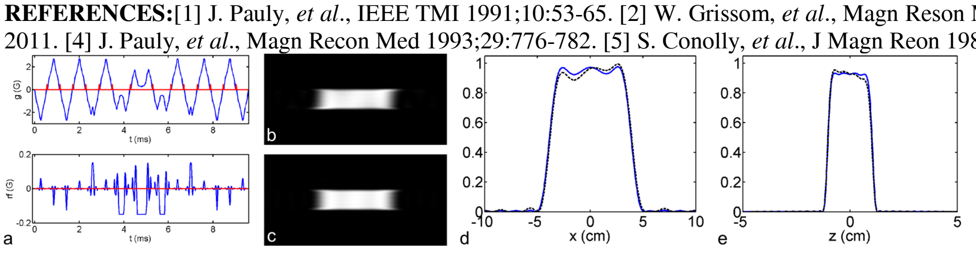

Fig. 2 a: A 2D refocusing RF pulse design by the proposed method (10% passband ripple, 1% stopband ripple, the VERSE method [5] is used to reduce the peak RF value). **b** and **c:** Excitation pattern by the method in [4] and the proposed method. **d** and **e:** Excitation profiles along the x- and z-axis (the black-dashed line: the method in [4]; the blue-solid line: the proposed method)