Minimum squared error estimate of electrical properties from B1 maps

Selaka Bulumulla[1], Seung-Kyun Lee[1], and Ileana Hancu[1]
[1]GE Global Research, Niskayuna, NY, United States

Target audience: Research and development personnel interested in measurement of tissue electrical properties for pathology, calculation of specific absorption rates (SAR) and treatment planning for hyperthermia using radio frequency (RF) fields.

$$\varepsilon_r = -\frac{\sum_{(m,n)\in S} Re\left(\left(\nabla^2 B1(m,n)\right)\cdot B1(m,n)^*\right)}{\omega^2 \mu \varepsilon_0 \sum_{(m,n)\in S} B1(m,n)\cdot B1(m,n)^*} \quad (1)$$

$$\sigma = \frac{\sum_{(m,n)\in S} Im\left(\left(\nabla^2 B1(m,n)\right)\cdot B1(m,n)^*\right)}{\omega \mu \sum_{(m,n)\in S} B1(m,n)\cdot B1(m,n)^*} \quad (2)$$

Figure 1 Minimum squared error estimate of permittivity and conductivity

Purpose: Extracting tissue electrical properties from transmit field maps (B1 maps) is a promising method with potential in-vivo applications[1]. However, the quality of images is impaired by noise in B1 maps. In previous work, this has been addressed by discarding electrical properties that are non-physical[2] (e.g. negative results) or filtering or smoothing B1 maps[3]. However, discarding non-physical values lead to missing pixels and filtering B1 may remove underlying spatial variations that are essential to estimate electrical properties. Therefore, we consider a minimum squared error approach to estimate conductivity and permittivity in order to obtain a robust calculation method.

Methods: The transmit field B1 and the electrical properties (conductivity and permittivity) satisfy the following equation at pixel(m,n): $\nabla^2 B1(m,n) + \left(\omega^2\mu\varepsilon(m,n) - j\omega\mu\sigma(m,n)\right)B1(m,n) = 0$. Consider a region S of constant electrical properties and define a cost function $\sum_{(m,n)\in S}\left(\mathrm{E}_{m,n}(\varepsilon,\sigma)\right)\left(\mathrm{E}_{m,n}(\varepsilon,\sigma)\right)^*$, where $\mathrm{E}_{m,n}(\varepsilon,\sigma) = \nabla^2 B1(m,n) + \left(\omega^2\mu\varepsilon(m,n) - j\omega\mu\sigma(m,n)\right)B1(m,n)$. Under ideal conditions, this cost function is zero. We find conductivity (σ) and permittivity (ε) as parameters that minimize this cost function. As the cost function is quadratic in ε and σ, the minimum is found by setting the first partial derivative to zero. The resulting equations are given in (1) and (2).

In order to validate the method with experiment data, a half sphere phantom was prepared with three spheres (2.5cm diameter) inside. The half sphere and the three spheres were filled with various concentrations of NaCl in distilled water. 1g/L of copper sulfate was added to all fluids. The composite phantom was imaged in a head coil (3.0T), B1 magnitude and phase data acquired and electrical properties were reconstructed using the proposed method. The Bloch-Siegert shift B1 mapping method was used to acquire B1 magnitude data and spin echo imaging was used to acquire B1 phase data [2]. The calculation omitted the pixels at the boundary of materials to avoid errors due to B1 transitions at material boundaries. The regions of constant properties were identified manually, using the spin echo image (magnitude) and knowledge of the phantom as a guide. The estimated conductivity and permittivity were compared with measured values (Agilent 85070E dielectric probe kit) at 200MHz.

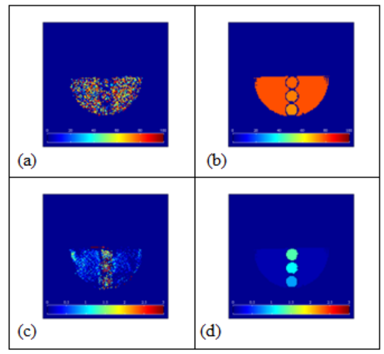

Figure 2 Permittivity pixel by pixel (a), proposed method (b), conductivity pixel by pixel (c), proposed method (d).

Results: The axial plane conductivity and permittivity images are shown in Fig. 2. The measured properties of the phantom fluids and the estimated properties are shown in Table 1.

Table 1 Measured and estimated properties of phantom fluids

Compartment	NaCl	Measured (200MHz)		Pixel by pixel		Proposed method	
		ε_r	σ	ε_r	σ	ε_r	σ
Outer	0.5 g/L	78	0.16 (S/m)	59	0.44(S/m)	79	0.13 (S/m)
Top	9 g/L	77	1.45 (S/m)	46.7	1.39(S/m)	77	1.33 (S/m)
Center	5 g/L	77	0.87 (S/m)	52.2	1.54(S/m)	75	1.12 (S/m)
Bottom	3.7 g/L	75	0.65 (S/m)	51.9	1.45(S/m)	73	0.81 (S/m)

Discussion: The conductivity and permittivity values estimated with the proposed method agreed better with measured values compared to pixel by pixel calculation averaged over a region. In extending to in-vivo applications, T1 weighted images can be used to identify tissue types (hence regions of constant electrical properties) to aid in segmentation of image to compartments of constant electrical properties.

Conclusion: The promising results point to the feasibility of robust conductivity and permittivity image reconstruction with proposed method.

References: 1.Bulumulla SB, et. al.,. Breast permittivity imaging. ISMRM; 2012; Melbourne, Australia. 2.Bulumulla SB, et. al.,. Conductivity and permittivity imaging at 3.0 T. *Concepts in Magnetic Resonance Part B.* 2012;41B(1):13-21. 3.Katscher U, et. al.,. Determination of electric conductivity and local SAR via B1 mapping. *IEEE TMI.* Sep 2009;28(9):1365-1374.