A COMPUTATIONAL MODEL OF THE TIME – INDEPENDENT BLOCH NMR FLOW EQUATION FOR MOLECULAR IMAGING

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INTRODUCTION

A new magnetic resonance methodology based on Bloch NMR flow equation and Hermite equations for detailed studies of processes taking place at molecular level in living tissues has been developed.

THEORETICAL FORMULATIONS

We study the flow properties of the modified time independent Bloch NMR flow equations which describes the dynamics of fluid flow under the influence of rF field [1, 2].

$$\frac{d^2M_y}{dx^2}+\frac{1}{vT_0}\frac{dM_y}{dx}+\frac{S(x)}{v^2}M_y=\frac{M_0\gamma B_1(x)}{v^2T_1} \quad \left(\text{where } S(x)=\gamma^2B_1^2(x)+\frac{1}{T_1T_2},\ \frac{1}{T_0}=\frac{1}{T_1}+\frac{1}{T_2}\right) \qquad (1)$$

Subject to the following conditions as highlighted in earlier works [1, 2]. T_1 and T_2 are the spin-lattice and spin-spin relaxation times respectively, the reciprocals of T_1 and T_2 are defined as relaxation rates. RF B_1 is the spatially varying magnetic field and v is the fluid flow velocity. However, when the rf $B_1(x)$ field is applied, M_y has a maximum value when rf $B_1(x)$ is maximum and $M_o = 0$. At the point when maximum NMR signal is received (maximum values of M_y and $B_1(x)$ respectively), equation (1) becomes:

$$\frac{d^2M_y}{dx^2}+\frac{1}{vT_0}\frac{dM_y}{dx}+\frac{S(x)}{v^2}M_y=0 \qquad (2)$$

We apply a fundamental transformation procedure given as $M_y(x)=\psi(x)e^{\lambda x}$ and provided that $\lambda=\frac{1}{2vT_0}$, the above equation becomes:

$\frac{d^2\psi(x)}{dx^2}+\frac{1}{v^2}\left(T_g-T_R-\gamma^2G^2x^2\right)\psi(x)=0$; where $T_g=\frac{1}{T_1T_2}$ and $T_R=\frac{1}{4T_0^2}$ and G is the strength of the gradient field. With further assumptions: $\gamma B_1(x)=i\gamma Gx$;

$\alpha=i\sqrt{\frac{\gamma G}{v}}x$; we have: $\frac{d^2\psi}{d\alpha^2}+\left(\frac{T_g-T_R}{\gamma Gv}-\alpha^2\right)\psi=0$. If we make another transformation as follows: $\psi(\alpha)=M_y(\alpha)e^{-\frac{\lambda}{i}\sqrt{\frac{v}{\gamma G}}\alpha}$ and $\frac{\lambda}{i}\sqrt{\frac{v}{\gamma G}}=\frac{\zeta\alpha}{2}$, we have:

$\frac{d^2M_y}{d\alpha^2}-2\alpha\frac{dM_y}{d\alpha}+2nM_y=0$ and given that $\left.\begin{array}{l}T_g-T_R=2n\gamma Gv\\ \zeta\approx 1\end{array}\right\}$, the final solution becomes

$$M_y(x)=H_n\left(i\sqrt{\frac{\gamma G}{v}}x\right)=(-1)^n\exp\left(-\frac{\gamma G}{v}x^2\right)\frac{d^n}{dx^n}\exp\left(-\frac{\gamma G}{v}x^2\right) \qquad (4)$$

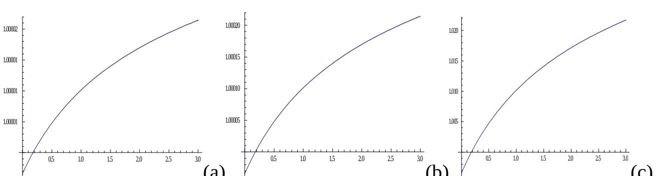
where $n=\frac{T_R-T_g}{2\gamma Gv}$.

ANALYSIS OF RESULTS

From the above expressions, we see that $1/\gamma GxT_0$ and if we therefore draw a simple analogy from short gradient pulse (SGP) in the pulsed-field gradient (PFG) NMR, we see that provided that the relation times T_1 and T_2 are properly chosen To represent the gradient pulse duration, the term $1/\zeta$ represents the phase change of the spin at the position x.

CONCLUSION

It would be observed from the illustrations given in Figures (1) and (2) that as the fluid velocity reduces as often encountered in cellular process, we see that the imaging equation as given in equation (4) shows contrast in terms of MR signals. Figure (2) shows that the behaviour of the MR signals is completely different for different tissues. Finally, it is quite interesting to note that the magnitude of the signals becomes so large at this level and hence, we may be able to follow processes at molecular level in real time in which we do not need to worry about blur images.



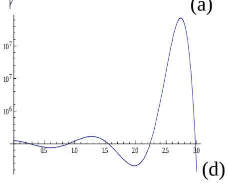


Fig.1: Plots of the transverse magnetization M_y (vertical axes) against the absolute (positive) values of α (horizontal axes) using the relaxations time – values of kidney at 1.5T[2], G = 10mT/m, $\gamma = 42.5781 \times 10^6$/T/s for (a) v = 3.0m/s (b) v = 0.3m/s (c) v = 0.003m/s (d) v = 0.000003m/s.

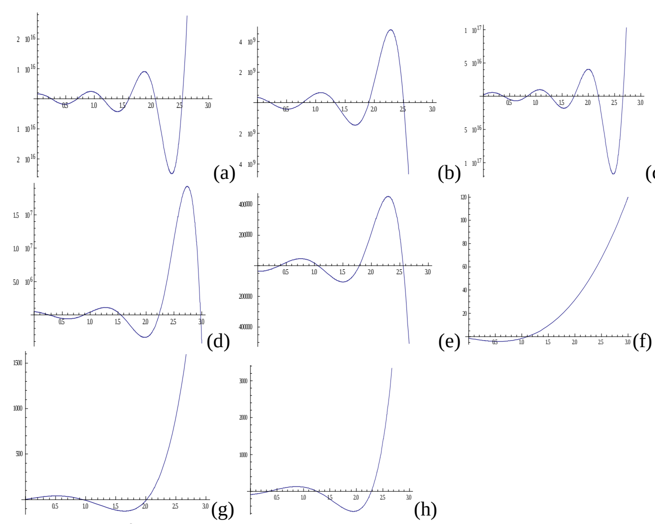


Fig. 4: Plots of the transverse magnetization M_y (vertical axes) against the absolute (positive) values of α (horizontal axes); G = 10mT/m, $\gamma = 42.5781 \times 10^6$/T/s, v = 0.000003m/s using the relaxations time – values, at 1.5T[2], of (a) skeletal muscle *(b) heart muscle (c)liver (d) kidney (e) spleen (f) fatty tissue (g) gray brain matter (h) white brain matter.*

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