# Image Reconstruction of Single-Shot North West EPI Data acquired with PatLoc Gradients using Magnetic Field Monitoring and Total Generalized Variation – Conjugate Gradient

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**Introduction:** The PatLoc (Parallel Imaging Technique using Localized Gradients) method uses nonlinear, nonbijective spatial encoding magnetic fields in addition to the three linear encoding fields for image encoding.[1,2] Single shot North-West Echo Planar Imaging (NW-EPI) [3] is a multidimensional trajectory designed to improve the resolution in a selected region of the image (e.g. top-left) by exploiting the spatially varying resolution characteristic of nonlinear encoding fields. Reconstruction of PatLoc data is usually done by iterative inversion of the encoding matrix. Total Generalized Variation (TGV) was recently introduced [5] and successfully applied to PatLoc imaging with 2D trajectories and showed promising results in terms of image quality, by reducing Gibbs ringing, undersampling artifacts and noise.[6] This work aims at utilizing the properties of TGV to improve the image quality of NW-EPI data. However, when reconstructing with multidimensional trajectories, TGV converges very slowly and is too time-consuming for practical use. We therefore propose a new concept of numerically solving the inverse problem, called Total Generalized Variation – Conjugate Gradient (TGV-CG) to increase convergence speed.

**Methods:** The gradient waveforms of the NW-EPI were computed by solving an optimization problem to match the local k-space trajectory in the region of interest to a target trajectory.[3] The target trajectory in this work was an EPI sequence covering twice the k-space extent of that achievable in given time using linear gradients alone. The target trajectory consists of 64 lines with 64 readout points each line for a total time of 41.6ms. The optimization was solved subject to peak gradient and slew rate constraints. The image is reconstructed on a pixel grid that is dense enough ($192^2$) to ensure that also the high local image resolution in the top-left region of the image is fully reflected. The trajectories were estimated using a dynamic field camera [3,4,5] consisting of 16 $^1$H field probes [5] approximatively distributed on a spheroids' surface ($\varnothing x \approx \varnothing y \approx 18$ cm and $\varnothing z \approx 14$ cm) operated in transmit/receive mode and placed inside the head coil. The probes were connected to the spectrometer of the scanner and controlled by an external trigger signal. The field probes' phases are reflecting the magnetic field evolution at their respective positions. The trajectory can then be estimated in the least square sense from the unwrapped field probe phases taking into account their position and their off-resonance frequency. As a model real spherical harmonics or a set of basis functions including Maxwell terms' spatial dependencies is used. The estimated k-space trajectory is used in the image reconstruction [5] in order to take into account effects such as eddy currents, field drifts and gradient delays which affect the image quality.

MR image reconstruction is defined as an inverse problem. In this work, the second-order TGV is used as regularization term:

$$\min_u \|\mathcal{F}(u) - g\|_2^2 + \alpha \cdot \mathrm{TGV}^2(u)$$

where $F$ is the forward operator, g the data affected by noise, $\alpha$ the regularization parameter and $\mathrm{TGV}^2$ the second-order TGV functional. The expression can be minimized by reformulating the problem as a saddle-point problem and employing abstract primal-dual (PD) algorithms.[8] Note that the concrete method depends on the saddle-point formulation and dualizing the discrepancy $\|F(u)-g\|^2_2$ has proven to work well [8] for TGV-minimization. However, as this results in too many iterations here, we choose not to dualize the discrepancy: the concrete primal-dual method then involves, in each iteration step, the solution of a linear equation. This is done with a conjugate gradient method, hence giving the method the name TGV-CG. The datasets were reconstructed on a $192^2$ image grid with 300 PD iterations and an experimentally found $\alpha$, depending on the dataset, on a NVIDIA GTX480 graphics card. A GPU-accelerated implementation makes the method practical, as it decreases the reconstruction time by over two orders of magnitude compared to a sequential MATLAB implementation. The data were also reconstructed with a conventional CG method with 50 iterations for comparison.

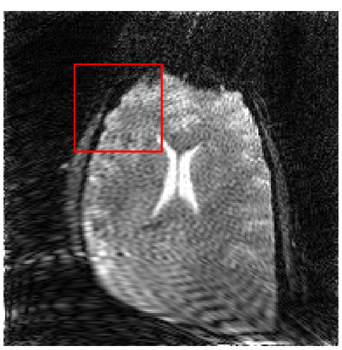

*Fig. 1: Reconstructed image with 50 CG iterations. The ROI is illustrated by a red rectangle.*

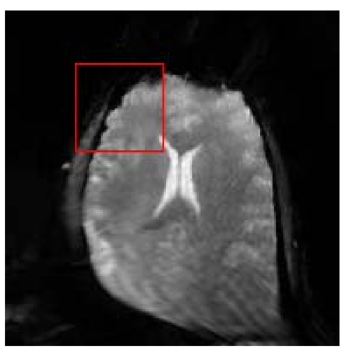

*Fig. 2: TGV-CG ($\alpha = 1.2$) regularized reconstruction with 300 iterations.*

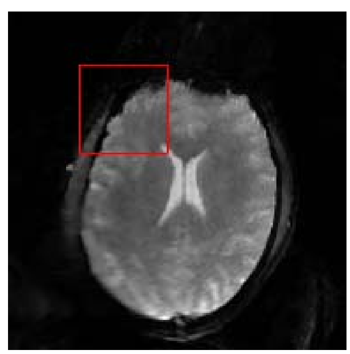

*Fig. 3: Taking the Maxwell terms in trajectory estimation and reconstruction into account. TGV-CG with $\alpha = 0.9$.*

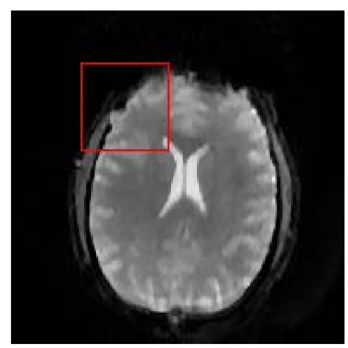

*Fig. 4: Conventional EPI, TGV-CG regularized ($\alpha = 1.2$)*

**Results:** Fig. 1 shows the image when reconstructed with a conventional CG method, which took approximately 37 seconds. The TGV-CG regularized reconstruction (Figs. 2-4) needs approximately 34min. The inner CG iterations of each PD iteration converge in 4 to 9 iterations leading to a total of approximately 2000 applications of the forward operator and its adjoint, depending on the given dataset and $\alpha$. The distortions in the bottom-right in Figs. 1 & 2 can be overcome by taking into account the Maxwell terms in the trajectory estimation and image reconstruction (Fig. 3). NW-EPI (Figs. 1-3) exhibits improved resolution within the ROI compared to an EPI acquired with conventional linear gradients and trajectory measured with the dynamic field camera (Fig. 4).

**Discussion and Conclusion:** For the evaluation of the image quality, the focus is on the top-left (red rectangle in all figures) because this area has the highest resolution. Areas outside this region of interest, specifically the bottom-right, are not expected to deliver good resolution. Figs. 2 & 3 exhibit better quality in this ROI and additionally reduce the artifacts outside the object. Besides the longer reconstruction time, TGV-CG proved to work very well in combination with measured trajectories. In contrast, the primal-dual algorithm applied with a conservative $\alpha = 10^{-7}$ did not converge even after 60,000 iterations (12 hours). Another advantage of TGV-CG is that the inner CG is initialized with the current image which can lead to faster convergence of the inner CG after a few PD iterations. We have shown that improvements of the image quality can be achieved with TGV-CG at the expense of increased reconstruction time. For future work, this could be improved by more sophisticated solvers for saddle-point problems.

**References:** [1] Hennig et al., MAGMA 21:5-17 (2008); [2] Schultz et al., MRM 64:1390-1403 (2010); [3] Barmet, Proc. ISMRM 2010, p.216; [4] Layton et al, MRM (in press, 2012); [5] Wilm B. et al., MRM 65:1690-1701 (2011); [6] Bredies et al., SIAM J. Imag. Sci., 3(3):492-526 (2010); [7] Knoll et al., MRM (in press, 2012); [8] Chambolle et al., J. Math. Imaging Vision, 40:120-145 (2011)