L1-Regularized GRAPPA Kernel Estimate

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Introduction: Compressed sensing utilizing L1-regularization has been established as a valid MRI reconstruction approach [1]. The L1-regularization is usually applied in the wavelet transform domain of image space. Traditional parallel imaging techniques utilizing L2 regularization, e.g. GRAPPA [2], are still used in clinical practice because of their high level of robustness. In this abstract, we propose a novel incorporation of L1-regularization for improved GRAPPA reconstruction. Instead of applying the L1-regularization in image space, we apply it in k-space to improve the GRAPPA kernel estimate. This is a novel way to automatically reduce the GRAPPA kernel support (a.k.a. GRAPPA kernel size) by assuming that the GRAPPA kernel should be compact in k-space. The new approach was implemented and used to reconstruct phantom images, and the reconstruction fidelity was compared to standard GRAPPA. The robustness was also studied by varying the L1-regularization parameter over 6 orders of magnitude.

Method: The GRAPPA kernel is typically estimated using the data from a set of fully sampled k-space lines, i.e. ACS lines. The GRAPPA kernel estimation can be implemented as either a linear regression problem, or an optimization problem. We use the latter implementation here. In order to prevent noise amplification due to the high condition number, the L2 norm regularization is often used:

$$\|Gk - b\|_2 + \lambda\|G\|_2 \quad (1)$$

where G is the GRAPPA kernel, k and b are the acquired and un-acquired k-space data point in ACS lines, λ is the L2 regularization strength.

It is accepted that a larger GRAPPA kernel can improve the accuracy of GRAPPA model but degrade the image SNR by introducing high level noise [4]. While the standard GRAPPA implementation uses a rectangular kernel in k-space, there is no simple way to adaptively select an optimal kernel size and shape. It is well-accepted that GRAPPA only needs a relatively small kernel in k-space (e.g., 4 x 5), which hints at its compact nature. Therefore, we propose to replace the L2-regularization in Eq. (1) by L1-regularization to promote the compactness of GRAPPA kernel:

$$\|Gk - b\|_2 + \lambda\|G\|_1 \quad (2)$$

Eq.(2) is solved using iterative majorization-minimization algorithm [3].

MR image data were acquired in one standard water bottle phantom to compare the L1-regularized GRAPPA kernel estimate method against a conventional L2-regularized GRAPPA kernel estimation. We acquired one SSFP real-time cine image series on a 1.5T MR scanner (MAGNETOM Avanto, Siemens Healthcare, Germany) using a 32-channel phased-array receiver coil (Rapid MR International, Columbus, OH). Imaging parameters were: 160 × 120 matrix, 8 mm thick slice, flip angle= 70^o, TE/TR = 1.0/2.4 ms, pixel bandwidth=1488 Hz/pixel, FOV = 400 × 300 mm^2. A total of 81 images per series with fully sampled k-space were acquired to support statistical analysis. All images were reconstructed offline on a personal computer (Intel Duo Core Quad 3.0GHz CPU, 16GB memory) using software written in MATLAB.

The GRAPPA kernel size was chosen to be 4 × 11. Both L2-regularized GRAPPA kernel and L1-regularized GRAPPA kernel were computed using the first fully-sampled frame as the ACS lines. The remaining frames were uniformly down-sampled by a factor of 6 and GRAPPA reconstructions were performed using both kernels. The root-mean-square error (RMSE) between the reconstructed k-space and the corresponding fully-sampled k-space was evaluated as the metric of reconstruction fidelity [4].

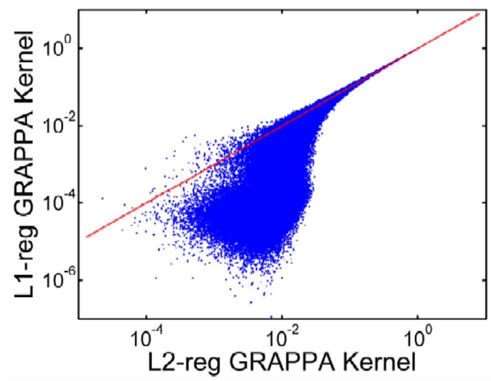

Figure 1. L2-regularized GRAPPA kernel vs L1-regularized GRAPPA kernel. The L1 regularization strength λ = 10^3. The dashed line indicates relation: Y = X. Each kernel has 4x11x32 coefficients.

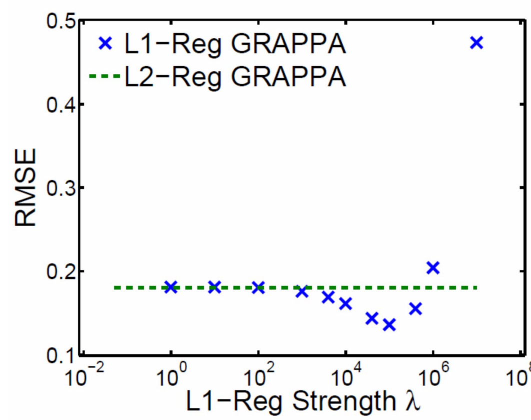

Figure 2. GRAPPA reconstruction RMSE vs. L1 regularization strength.

Results: Figure 1 shows the comparison between the absolute value of the L2-regularized GRAPPA kernel and the L1-regularized GRAPPA kernel. The large coefficients are identical, while a portion of small coefficients are suppressed more than 10 times in L1-regularized GRAPPA kernel. Figure 2 shows the GRAPPA reconstruction RMSE vs. L1 regularization strength. Over a wide range of regularization strength ($10^2 < \lambda < 10^6$), a positive RMSE gain was observed.

Discussion and Conclusion: We demonstrate the L1 regularization can be effectively used in GRAPPA kernel estimation to promote the GRAPPA kernel sparsity in k-space. Compared to the standard L2-regularized GRAPPA kernel, the L1-regularized kernel preserves high amplitude coefficients, while selectively suppressing low amplitude coefficients. The phantom results generated using the L1 regularized GRAPPA kernel demonstrated lower RMSE over a wide range of regularization strengths. Therefore the new GRAPPA kernel estimate has higher fidelity over the traditional GRAPPA kernel estimate relatively independent of strength of regularization. It provides a simple and novel solution to improve GRAPPA reconstruction, and potentially could benefit the clinical practice.. In this implementation, the computation time is still significantly longer than the L2-regularized algorithm; however, because the kernel size is small, it is still much shorter than the computation of L1-regularization in image space. It should be noted that when the regularization parameter is large, (> 10^5), the artifact level increases even while the RMSE was still low. Future studies are warranted to optimize the regularization parameter.

References: [1] Lustig M, et al, Magn Reson Med, 58 (2007), 1182. [2] Pruessmann, KP et al, Magn Reson Med 42 (1999) 952. [3] Kragh TJ and Kharbouch AA, in Proceedings, IEEE ICIP, Atlanta, GA, Oct. 2006, 645. [4] Nana R et al, Magn Reson Med 59: (2008), 819