

# Accelerated MRI through Aliased and Sub-sampled $k$ -space acquisitions

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## INTRODUCTION

A new multi-coil MRI data reconstruction method for restoring aliased and sub-sampled  $k$ -space data to achieve a multiplicative increase in scan acceleration is presented. Recently, the RATE [1] method for accelerating dynamic MRI scans was introduced, in which, a signal excitation module built from RF pulses and gradients, was inserted into an EPI sequence to alias blocks of  $k$ -space phase encodes. The acquired  $k$ - $t$  space dataset was then un-aliased during reconstruction through a Fourier transformation along the time axis. In this work, it is first shown that for both normal and dynamic MRI scans, RF coil sensitivity profiles can be used to resolve aliased  $k$ -space samples. Secondly, if the aliased  $k$ -space is also simultaneously sub-sampled, a Parallel Imaging (PMRI) [2] algorithm is used as a first reconstruction step to synthesize the un-acquired aliased  $k$ -space samples. Next, the coil sensitivity profiles are used again with the acquired and PMRI synthesized aliased  $k$ -space samples in a second, iterative reconstruction step to obtain the fully restored  $k$ -space dataset. The proposed method can enable acceleration factors as high as 12 for 2D scans as is demonstrated using *in vivo* data.

## THEORY

The signal encoding process for aliased  $k$ -space acquisitions with RATE is represented by:

$$\mathbf{E}_{(n,l),\rho} = \left[ \sum_{m=1}^{\mathbf{R}_k} \mathbf{a}_m e^{i\mathbf{k}_n^m \mathbf{r}_\rho} \right] \mathbf{C}_1(\mathbf{r}_\rho) \quad [1]$$

Here,  $\mathbf{r}_\rho$  denotes the  $\rho^{\text{th}}$  voxel location,  $\mathbf{R}_k$  is the  $k$ -space aliasing factor,  $\mathbf{k}_n^m$  is the  $m^{\text{th}}$  sample at the  $n^{\text{th}}$  location in aliased  $k$ -space,  $\mathbf{C}_1$  is the spatial sensitivity profile of the  $l^{\text{th}}$  receiver and  $\mathbf{a}_m$  are the amplitudes of the FID signal pathways derived using the Bloch equations. Accordingly, the aliased acquisition is given by:

$$\mathbf{E}\mathbf{v} = \mathbf{d} \quad [2]$$

Here,  $\mathbf{v}$  is the desired image vector and  $\mathbf{d}$  is the acquired aliased  $k$ -space data from all receivers. Since receiver sensitivity profiles are globally smooth functions with limited support in  $k$ -space,  $\mathbf{E}$  could be poorly conditioned depending on the size of the aliased  $k$ -space phase encode blocks. Therefore, the general framework for solving eqn.2 is:

$$\mathbf{v}_{\text{reg}} = \underset{\mathbf{v}}{\text{argmin}} \left( \|\mathbf{E}\mathbf{v} - \mathbf{d}\|_2^2 + \sum_{i=1}^N \lambda_i \varphi_i(\mathbf{v}) \right) \quad [3]$$

Here,  $\varphi_i(\mathbf{v})$  are regularization functions and  $\lambda_i$  determine the extent of the constraint imposed by  $\varphi_i(\mathbf{v})$  on the solution. When aliased  $k$ -space data has also been sub-sampled, the first step in the reconstruction process is to synthesize the un-acquired aliased samples using PMRI. However, the PMRI synthesized aliased samples cannot be used directly as substitutes, in eqn.3, for the un-acquired data points since they are not error-free estimates of the same and can adversely impact output image quality. Therefore, the encoding matrix  $\mathbf{E}$  is split to utilize the acquired and PMRI synthesized samples separately. Now, the solution to eqn.2 is:

$$\mathbf{v}_{\text{reg}} = \underset{\mathbf{v}}{\text{argmin}} \left( \left\| \begin{bmatrix} \mathbf{E}_k & \mathbf{0} \\ \mathbf{0} & \mathbf{E}_{\text{pmri}} \end{bmatrix} \mathbf{v} - \begin{bmatrix} \mathbf{d}_k \\ \mathbf{d}_{\text{pmri}} \end{bmatrix} \right\|_2^2 + \lambda_p \left\| \begin{bmatrix} \mathbf{E}_k & \mathbf{0} \\ \mathbf{0} & \mathbf{E}_{\text{pmri}} \end{bmatrix} \mathbf{v} - \begin{bmatrix} \mathbf{d}_k \\ \mathbf{d}_{\text{pmri}} \end{bmatrix} \right\|_2^2 + \sum_{i=1}^N \lambda_i \varphi_i(\mathbf{v}) \right) \quad [4]$$

Here,  $\mathbf{d}_k$  contains the acquired aliased samples,  $\mathbf{d}_{\text{pmri}}$  contains the PMRI synthesized aliased samples,  $\mathbf{E}_{\text{pmri}}$  is the signal encoding matrix for the PMRI synthesized aliased samples, and  $\mathbf{E}_k$  is the signal encoding matrix for the acquired aliased  $k$ -space samples. For a particular PMRI acceleration factor  $\mathbf{R}_{\text{pmri}}$ ,  $\lambda_p$  is chosen to be the smallest value that eliminates image domain aliasing artifacts. In order to select  $\varphi_1(\mathbf{v})$  for eqn.4, a first estimate of the desired image vector was generated from eqn.4 without using any penalty functions. It was found that this estimate consisted of a highly oscillatory artifact spread along the aliased dimension. An ideal regularization option for minimizing such an artifact is the Total Variation (TV) function. However, in this work, the quadratic upper bound [3] of the TV function was used to make the entire eqn.4 quadratic. Next, the gradient of eqn.4 was equated to zero and the resulting linear system of equations solved using the standard CG method [4], to obtain  $\mathbf{v}_{\text{reg}}$  efficiently.

## RESULTS

For a simulation study, a full T1 weighted spin echo brain dataset with TE/TR=12/750ms, slice thickness=5mm, matrix size=240×240, FOV =210 mm<sup>2</sup>, was acquired using a Siemens 3T scanner and a 32-channel head coil. For  $\mathbf{R}_k=3$ ,  $\lambda_{\text{TV}}=10^{-4}$ ,  $\lambda_p=0.45$  and an aliased block size of 80, the results are displayed in Fig.1. In addition to  $\mathbf{R}_k$ , for images (c) and (d),  $\mathbf{R}_{\text{pmri}}$  was equal to 2 and 4 respectively. The RMSE values for the three images were 0.026, 0.028 and 0.0287. Next, an accelerated cardiac triggered dataset ( $\mathbf{R}_k=3$ , block size=80,  $\lambda_{\text{TV}}=10^{-4}$ ,  $\lambda_p=0.45$ , FOV=300mm<sup>2</sup>), was acquired using a EPI sequence and a 20-channel cardiac coil. The flip angles in the signal excitation module were 10°, 21° and 10°. Fat saturation was used to remove off-resonance effects unrelated to the reconstruction. The aliased  $k$ -space was further decimated offline to simulate PMRI and then fully restored as shown in Fig.2. For R=12, reconstructing cardiac data, in MATLAB (Natick, USA) on a computer with an Intel 1.8GHz processor and 3 GB memory, took 34 seconds.

## CONCLUSIONS

Reconstruction performance depends on many factors such as coil geometry, aliased  $k$ -space block sizes and the signal excitation module specifications. The ideal relationship between these parameters for optimal reconstruction is a topic of current investigation.

REFERENCES: [1] Arunachalam et al. In proc. 20<sup>th</sup> ISMRM, 2012, p643. [2] Griswold et al. MRM.2002. 47(6):1202. [3] Figueiredo et al. In *IEEE International Conference on Image Processing, 2006* (pp. 2633-2636). [4] Stoer J et al. *Numerische Mathematik 2*, 3rd ed. P.296

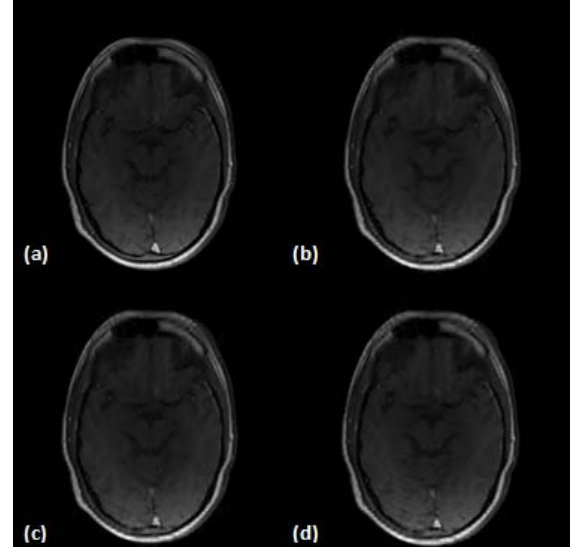


Figure 1: (a) Original Image. (b), (c) and (d) are reconstructed images for total acceleration factors R equal to 3, 6 and 12 (20 phase encodes) respectively.

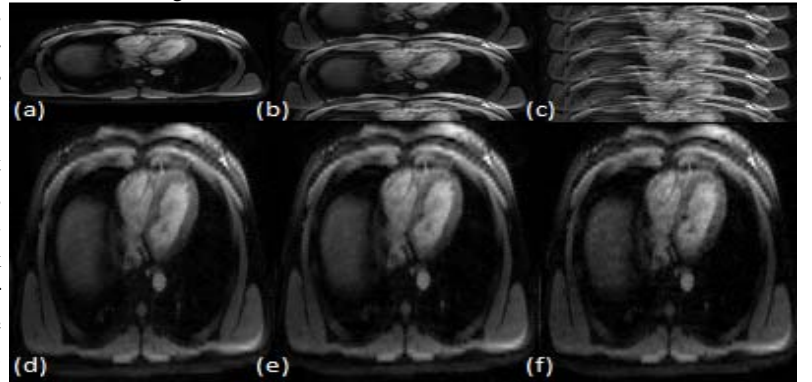


Figure 2: (a),(b), (c) are the uncorrected time frame images and (d), (e) and (f) are the restored images for total acceleration factor R equal to 3, 6 and 12 respectively.