Differential Energy: A k-space PPI Reconstruction Optimization Metric

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Target Audience This image assessment tool is suggested for clinicians and reserches seeking to quantify reconstruction fidelity when using k-space based Partial Parallel Imaging techniques.

Purpose Partial Parallel Imaging (PPI) allows faster scan times or improved resolution in clinical applications. The relative error (RE) or root-mean-squared error (RMSE) between a fully acquired reconstruction and the under-sampled one is often employed as a measure of the quality of the PPI algorithm [1-2]. Unfortunately, in a practical setting these are not possible to compute since a reference image is not obtained. Measures that do not require reference data, such as total variation (used heavily in sparse MRI applications [3]) or gradient energy [4] have been used to quantify image fidelity, but these measures are empirically based. In the GRAPPA algorithm [5] where missing lines of k-space are computed for each coil, a more direct quantification is possible. Taking advantage of the knowledge that the GRAPPA weights can be considered an operator (G) that can reproduce subsequent lines upon repeated application [6], one can recreate a copy of acquired lines using the calibrated weights for direct comparison (Figure 1).

The comparison is simply a complex product of the difference between the recreated signal, S_{rec}, and the acquired signal, S_{acq}, at each point in the line, summed over all available acquired lines. We call this quantity the Differential Energy (E_D), for the j^{th} coil:

$$E_D^j = \sum_{k_x,k_y \in S_{acq}} \left[S_{rec}^j(k_x,k_y) - S_{acq}^j(k_x,k_y)\right]\overline{\left[S_{rec}^j(k_x,k_y) - S_{acq}^j(k_x,k_y)\right]} \quad (1)$$

the overbar denotes the complex conjugate. The overall E_D is then the sum for all coils. We propose the E_D measure as a useful tool for optimizing GRAPPA algorithm parameters used to obtain the calibration weights and we demonstrate the effectiveness of the E_D for predicting the optimal region width in r-GRAPPA; a regionally optimized GRAPPA algorithm [7].

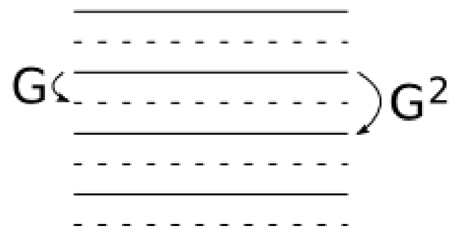

Figure 1 In a GRAPPA reconstruction for ORF=2, a single application of the G operator computes the missing lines, and a second application computes a copy of the existing acquired lines.

Methods Preliminary tests have been conducted on *in vivo* axial abdominal images acquired on a 1.5T GE scanner with an SPGR sequence using a 4-element coil array, 5mm slice thickness, TR/TE = 120/2.12ms and a 256x256 matrix size. For each coil, a partial dataset was sampled from full k-space to simulate an accelerated acquisition. The missing data was recovered using the r-GRAPPA algorithm and the region width (RW) parameter was optimized using both a RE comparison with the fully acquired reconstruction and with the E_D. The GRAPPA reconstruction kernel used 4 blocks and 16 auto-calibration-signal (ACS) lines (as in [5]). Distribution of error is represented in a difference map between accelerated and full k-space root sum-of-squares (rSoS) images. The optimized r-GRAPPA images were assessed using RE and E_D as well as visual inspection of resultant images and difference maps. For simplicity, an outer reduction factor (ORF) of 2 was used, but the results can be extended to higher ORFs.

Results Fig. 2 shows results for r-GRAPPA reconstructions with ORF=2. The minima of the RE and E_D curves consistently predict similar optimal RW for reconstruction (Fig. 1d). When optimized by the RE measure, the optimal RW is 4 pixels; for E_D the optimal RW is 24 pixels. Although there is a discrepancy in the optimized RWs, the difference maps (Fig. 2e&f) show similar level of error for both reconstructions and the residual aliasing artifact is virtually equivalent in the reconstructed images (Fig. 2b&c). The increase in RE and E_D at very low RW is the result of noise contaminating the fitted weights.

Discussion Optimization of r-GRAPPA RW is achieved using the reference free Differential Energy measure. The E_D predicts an RW similar to that indicated by RE comparisons using the fully sampled reference image. The E_D provides an effective tool for balancing the trade off between optimal reconstruction RW and influence of noise on the calibration step.

Conclusion The E_D metric may be useful for optimizing reconstruction parameters in any algorithm in which missing k-space lines are computed for each coil (*i.e.* see for example [1, 2, 8, 9]). It is suitable in situations where no reference image is available for comparison.

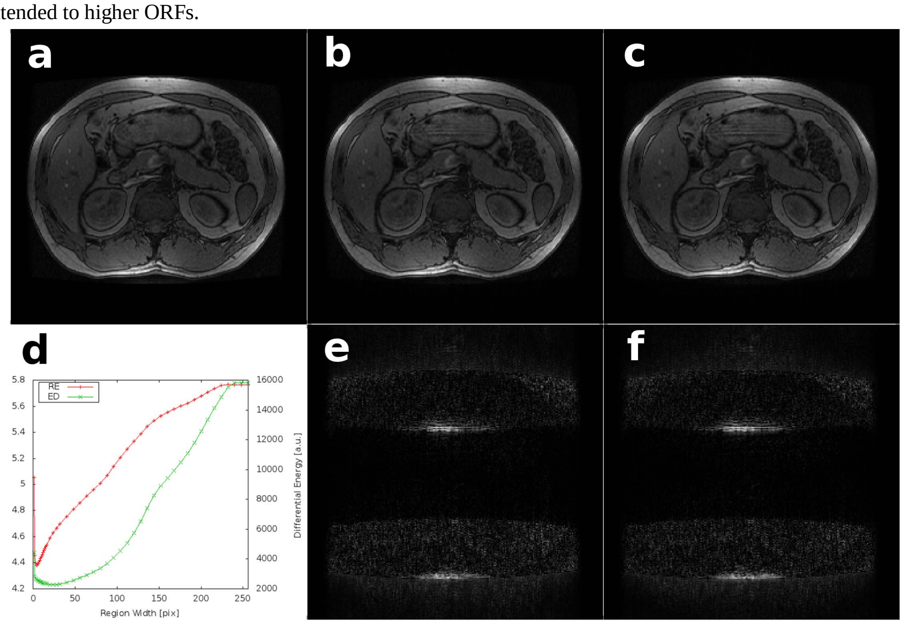

Figure 2 (**a**) Full k-space reference image. (**b**) r-GRAPPA reconstruction and (**e**) difference map with RE predicted optimal RW (4 pixels); (**c**) r-GRAPPA reconstruction and (**f**) difference map with ED predicted optimal RW (24 pixels); (**d**) RE and ED for r-GRAPPA reconstructions with various RW's (RW=256 corresponds to standard GRAPPA).

References

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