Denoising in Parallel Imaging via Structured Low-rank Matrix Approximation

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Introduction: Parallel imaging is a robust acceleration technique based on extracting spatial information from subsampled and noisy multi-channel observations. Interpolation approaches such as GRAPPA [1] use correlation across channels and between adjacent k-space blocks to estimate finite impulse response (FIR) interpolation filters from central k-space (ACS) lines. However, they exhibit a noise amplification effect [2,3] that may be mitigated via regularization techniques including total-variation, ℓ_1 and ℓ_2 norms [3-5]. However, regularization can be computationally expensive and may not be effective in eliminating both the noise and ghost artifacts without blurring the images. Here, we propose a pre-processing technique based on structured low-rank matrix approximation via truncated singular value decomposition (TSVD), which is able to suppress noise and ghost artifacts efficiently. TSVD method has been previously used in parallel MRI to improve the conditioning of the system matrix [6] and to reconstruct k-space via matrix completion [7]. In contrast to previous work, here rank properties are used to denoise acquired data in a computationally simple preprocessing for GRAPPA reconstruction.

Theory: The calibration and reconstruction equations in GRAPPA can be written as $y_{acs} = A_{acs}g$ and $y_r = A_r g$ respectively, where A_{acs} and A_r are calibration and reconstruction matrices in Toeplitz-block-Toeplitz (TBT) form, and g represents interpolation filters. Since the coil sensitivities are smooth, they can be well-approximated by FIR filters of size (m_h, n_h) in 2D k-space. It can be proven that when $K > R + 2$, there generically exist interpolation filters if A_{acs} (or A_r) has non-zero nullity equal to

$$Km_g n_g - (m_h + Rm_g - R)(n_h + n_g - 1) \qquad (1)$$

where K, R and (m_g, n_g) represent the number of coils, uniform downsampling factor applied only along m direction, and size of the interpolation filters, respectively. In addition, from ACS data we can create a matrix A_c that is a convolution operator formed from the ACS lines without downsampling; the nullity of the Toeplitz matrix A_c is equal to (1) with $R = 1$. As a result, matrices A_{acs}, A_r and A_c have low rank in the noise-free case. In practice, noise distorts the low-rank property of these matrices; however, they can be well-approximated by the closest low-rank matrix via singular value thresholding.

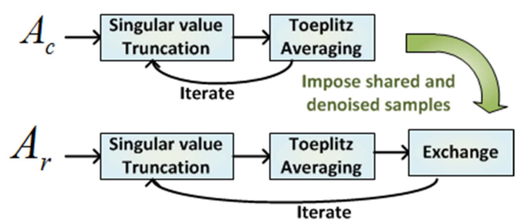

Figure 1: TSVD GRAPPA applies low-rank matrix approximation via iterative singular value thresholding and Toeplitz averaging.

Methods: In order to denoise the data, we used Cadzow's algorithm [8], which applies TSVD and TBT averaging iteratively, on A_c and on A_r sequentially (Step 1 and Step 2). From (1), A_c has lower rank than A_r and can be better approximated via TSVD; therefore, the shared samples are used in A_r for in Step 2 (see Fig. 1). As a result, both the ACS data and the uniformly subsampled data are jointly denoised. Then, the GRAPPA method is used to interpolate the denoised signal.

Results: Twelve coil-pMRI data were simulated from Biot-Savart law coil sensitivities (matrix size: 256x256); 4-fold downsampling was applied along one direction, and 18 central k-space lines (ACS) were acquired for an effective subsampling ratio of 3.12. As shown in Fig. 2, the GRAPPA reconstruction (kernel size 4x5x12) exhibits severe noise and ghost artifacts. The thresholds for TSVD were chosen using Bartlett's test [9]. Five iterations were sufficient. Fig. 2 shows that TSVD GRAPPA reconstructs the image with fewer artifacts than both GRAPPA and TV regularized GRAPPA [4] (here implemented using [10]); peak signal to noise ratios (PSNR) are given in Fig. 2.

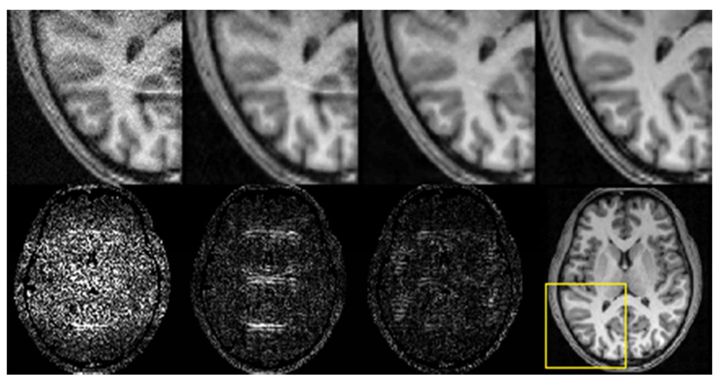

Figure 2: (From left to right) GRAPPA (20dB), TV regularized GRAPPA (27dB), TSVD GRAPPA (30dB) and original image. Top: Zoomed reconstructed images. Bottom: Difference images and the original image (rightmost).

Conclusions: We presented a pre-processing method based on low-rank approximation of structured matrices. The simple denoising step, used in conjunction with GRAPPA, reduces both noise and ghost artifacts without blurring.

References: [1] Griswold et al., MRM 2002; 47(6):1202-1210. [2] Breuer et al., MRM 2009; 62(3):739-746. [3] Lustig et al., ISMRM'09. [4] Zhang et al., ISMRM'10. [5] Weller et al., ISMRM'10. [6] Kyriakos et al., NMR BIOMED 2006; 19:379-392. [7] Lustig et al., ISMRM'10. [8] Cadzow, IEEE ASSP 1988; 36(1):49-62. [9] Bartlett, BRIT J STAT PSY 1950; 3:77-85. [10] Becker et al., "NESTA," 2009, Tech. Rep.