From Matrix to Tensor: Compressed Sensing dynamic MRI Using Tensor Based Sparsity

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Introduction: Compressed Sensing (CS) has been applied in dynamic Magnetic Resonance Imaging (dMRI) (1) to accelerate the data acquisition by exploring the sparsity of the signals. Conventionally, the 3D/4D datasets were separated into series of 2D images. The spatial and temporal information were then sparsified independently and sequentially. Therefore the spatial-temporal correlation may not be sufficiently exploited in the conventional approaches. In this work, we introduce the concept of tensor sparsity for CS-dMRI. Inspired by a recent application of 2D-SVD in CS-MRI (2), the Tucker model based Higher-order Singular Value Decomposition (HOSVD) (3) is applied in the CS-dMRI framework. Instead of treating the 3D/4D data as series of 2D images, HOSVD inherits the high-dimensional data format, leading to significantly improved dMRI reconstructions compared with those well-established CS-dMRI methods.

Theory: CS-dMRI framework can be summarized as: $Minimize: \|\Psi(\boldsymbol{M})\|_1 \; s.t. \|\Phi_F(\boldsymbol{M}) - y\|_2 \le \varepsilon$ [1], where Ψ and Φ_F denote the sparsity basis and the partial Fourier transform respectively; *y* is the *k*-space measurements; ε is the error tolerance; and $\boldsymbol{M}$ is the 3D/4D images. Conventionally, Ψ consists of a 2D transform (e.g. 2D wavelet transform) for in-plane sparsity and an extra transform (e.g. 1D Fourier transform) for temporal sparsity. This combination of subsequent vector and matrix operations can be ineffective in finding the sparse representation of the multi-dimensional datasets. In fact, there exists multi-dimensional sparsifying transforms, such as the Higher-order Singular Value Decomposition (HOSVD), that maintains the original data format and reduces the data redundancy by exploiting the information correlations in all dimensions. Briefly, any set of complex 3D images $\boldsymbol{M}$ can be decomposed as: $\boldsymbol{M} = \boldsymbol{S} \times_1 U_1 \times_2 U_2 \times_3 U_3$ [2], where $\times_n$ denotes the *n-mode* product of a tensor by a matrix. As demonstrated in (3), $U_n, n = 1,2,3,$ in equation [2] is the left singular matrix of the correlated *n-mode* matrix unfolding of tensor $\boldsymbol{M}$. Therefore, the computation of HOSVD in equation [2] leads to three different 2D-SVD operations: $S_n = U_n M_{(n)} V_n, n = 1,2,3$ [3], where $M_{(n)}$ denotes the *n-mode* matrix unfolding of tensor $\boldsymbol{M}$. The core tensor $\boldsymbol{S}$ can then be computed as $\boldsymbol{S} = \boldsymbol{M} \times_1 U_1^H \times_2 U_2^H \times_3 U_3^H$ [4]. With the unitary matrices $U_n, n = 1,2,3,$ obtained, we can then construct the tensor sparsifying transform as: $\Psi(\boldsymbol{M}) = \boldsymbol{M} \times_1 U_1^H \times_2 U_2^H \times_3 U_3^H$ Fig.1 shows the sparse representation, $\boldsymbol{S}$ of the 3D dataset $\boldsymbol{M}$ in HOSVD basis. It is clearly shown that the coefficients with large values are highly concentrated in one corner, while the vast majority of the elements in the *S* tensor are close to zero.

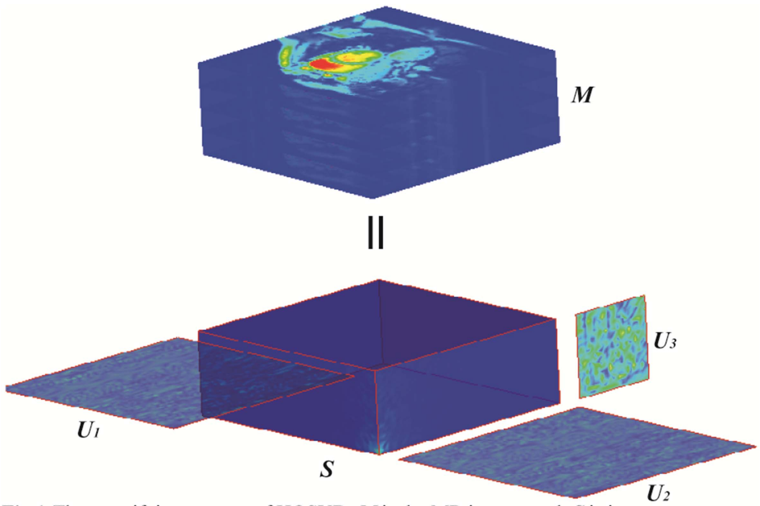

Fig.1: The sparsifying process of HOSVD. $\boldsymbol{M}$ is the MR images and, $\boldsymbol{S}$ is its sparse representation. It is noted that $\boldsymbol{S}$ is in its logarithmic scale to assist the presentation because $\boldsymbol{S}$ is too sparse to be easily visible. In $\boldsymbol{S}$, there is only very small area has high values (light blue), the rest is very close to zero (deep blue), showing the sparsity of HOSVD

Methods: 25 frames of full cardiac cine k-space sourced from (4) was acquired using a 1.5T Philips system with an in-plane resolution of 256 × 256. The data was obtained using a steady-state free precession (SSFP) sequence with a flip angle of 50 degree and TR = 3.45 msec. The FOV was 345 mm × 270 mm. The slice thickness was 10 mm. Each frame was randomly under-sampled in a retrospective fashion. The proposed method, k-t FOCUSS (4) and k-t SPARSE (5) were used for image recovery. k-t SPARSE employs 2D wavelet transform for in-plane sparsifying followed by the 1D Fourier transform along the temporal dimension. k-t FOCUSS uses 1D Fourier transform only for temporal sparsity. The results of all three methods were compared.

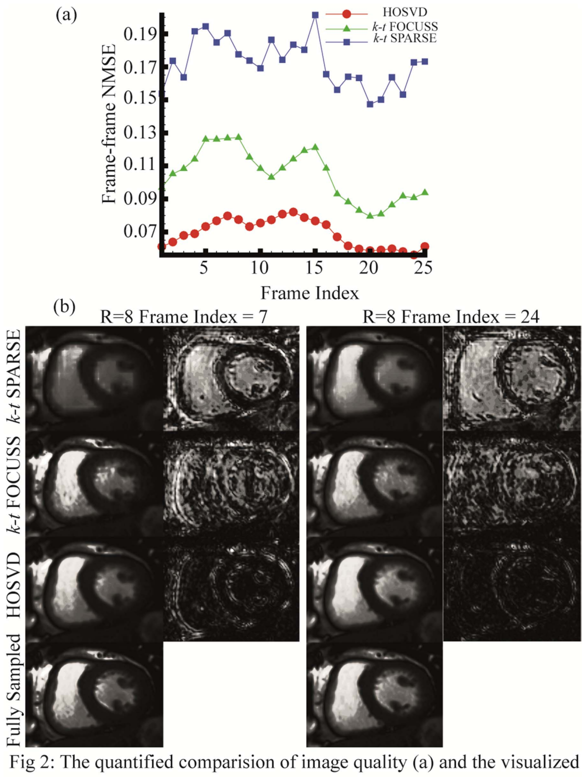

Fig 2: The quantified comparision of image quality (a) and the visualized comparison of image quality (b). It shows that the HOSVD outperformed the *k-t* FOCUSS and *k-t* SPARSE in the reconstruction accuracy.

Results and Discussion: Fig.2(a) quantifies the image quality of the proposed method compared to k-t FOCUSS and k-t SPARSE, in terms of frame by frame normalized MSE (NMSE) when the reduction factor was 8. For all the frames, the proposed method consistently provided lower NMSE than the conventional methods. Fig.2(b, left) and Fig.2(b, right) show the reconstructed frames and their error maps (enlarged 4 times), where the proposed method achieved the highest and lowest NMSE, respectively. Consistent with the quantified evaluation, HOSVD achieved better image contrast, highest resolution and lowest artefacts. In Figs. (1) and (2), HOSVD demonstrated its ability to simultaneously explore spatial-temporal sparsity and, therefore provide better reconstruction accuracy than the conventional methods.

Conclusion: This work proposed a novel concept of tensor sparsity for Compressed Sensing dynamic MRI, and presented the Higher-order Singular Value Decomposition as an example. The proposed HOSVD transform simultaneously sparsifies both temporal and spatial information, offering advantages of the tensor sparsity in terms of reconstruction accuracy, which have been demonstrated in a cardiac dynamic MRI study.

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Abstract: In this work, we introduce the conception of tensor sparsity for Compressed Sensing dynamic MRI. Conventionally, the spatial and temporal information were then sparsified independently and sequentially. Therefore the spatial-temporal correlation may not be sufficiently exploited. This work applys the Tucker model based Higher-order Singular Value Decomposition (HOSVD) in the Compressed Sensing dynamic MRI framework. Instead of treating the 3D/4D data as series of 2D images, HOSVD inherits the high-dimensional data format, leading to significantly improved dMRI reconstructions compared with those well-established CS-dMRI methods. The advantages of the tensor sparsity in terms of reconstruction accuracy have been demonstrated in a given cardiac dynamic MRI study.