Denoising Image Sequences: Algorithm and Application to Quantitative MR Imaging

Fan Lam[1,2], Bo Zhao[1,2], Michael Weiner[3,4], Norbert Schuff[3,4], and Zhi-Pei Liang[1,2]

[1]*Electrical and Computer Engineering, University of Illinois at Urbana-Champaign, Urbana, IL, United States,* [2]*Beckman Institute, University of Illinois at Urbana-Champaign, Urbana, IL, United States,* [3]*Center for Imaging of Neurodegenerative Diseases, Department of Veteran Affairs Medical Center, San Francisco, CA, United States,* [4]*Department of Radiology and Biomedical Imaging, University of California, San Francisco, CA, United States*

Introduction:

Denoising is widely used for various quantitative MR imaging applications (e.g., quantitative diffusion imaging [1], parametric mapping [2] and spectroscopic imaging [3]), where a sequence of images is acquired and used for parameter estimation. Although it is straightforward to perform spatial filtering of individual images [4-6], and/or temporal filtering of each voxel [7-9], it has been demonstrated that jointly denoising multiple images can improve performance, especially in low SNR regions [1, 10-11]. A typical approach to joint denoising (e.g., [1]) is to enforce the entire image sequence to have shared edge structures. However, the edges estimated from noisy data using this approach can be biased when there are images with distinct edges in the sequence. For example, Fig. 1 illustrates the correlation between the edges of a T2-weighted image sequence. As can be seen, the edges from neighboring frames are highly correlated but this correlation is gradually reduced for the more distant frames. We propose here a new denoising method, based on two modeling assumptions: (1) the edge structures in the image sequence are correlated and admit a low-rank representation; (2) the edge images are sparse/highly compressible. We use a penalized maximum likelihood (PML) estimation formalism to integrate these components and develop a computationally efficient algorithm to solve the associated optimization problem. The proposed method has been evaluated using simulated and experimental data and provided excellent denoising results for all the cases tested.

Proposed Method:

We consider the denoising model: $\mathbf{Y} = \mathbf{X} + \mathbf{N}$, where $\mathbf{Y}$ and $\mathbf{X}$ are NxQ matrices whose columns correspond to the noisy and noiseless images, respectively. Assuming complex white Gaussian noise for $\mathbf{N}$, we propose to denoise $\mathbf{Y}$ using the following formulation

$$\hat{\mathbf{X}}, \hat{\mathbf{P}}, \hat{\mathbf{Q}} = \arg\min_{\mathbf{X},\mathbf{P},\mathbf{Q}} \|\mathbf{Y} - \mathbf{X}\|_F^2 + \alpha \|\mathbf{D}\mathbf{X} - \mathbf{P}\mathbf{Q}\|_F^2 + \lambda\alpha\Phi(\mathbf{P}\mathbf{Q}). \quad (1)$$

$\mathbf{D}$ is an operator that computes finite differences of each image along all D dimensions. $\mathbf{P} \in \mathbb{C}^{DNxL}$ and $\mathbf{Q} \in \mathbb{C}^{LxQ}$ are low-rank decomposition [12-15] of the edge images. $\Phi(\mathbf{PQ})$ is a sparsity-promoting function. Here, we use $\Phi(\mathbf{PQ}) = \|\mathbf{PQ}\|_1$. The penalty parameters α and λ are used to trade off the data consistency, the low-rank penalty and the sparsity constraint. We design an alternating minimization algorithm to solve the problem in (1). Specifically, for fixed $\hat{\mathbf{X}}$, we update $\mathbf{P}$ and $\mathbf{Q}$ by solving

$$\hat{\mathbf{P}}, \hat{\mathbf{Q}} = \arg\min_{\mathbf{P},\mathbf{Q}} \left\|\mathbf{D}\hat{\mathbf{X}} - \mathbf{P}\mathbf{Q}\right\|_F^2 + \lambda\|\mathbf{P}\mathbf{Q}\|_1. \quad (2)$$

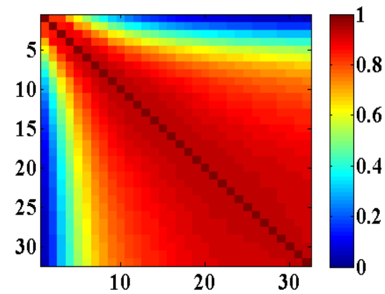

Fig. 1: Correlation matrix for the edges from a T2-weighted image sequence.

Equivalently, we introduce an auxiliary variable $\mathbf{S} = \mathbf{PQ}$ and use the augmented Lagrangian method (ALM) [16-17] to solve

$$\hat{\mathbf{P}}, \hat{\mathbf{Q}}, \hat{\mathbf{S}} = \arg\min_{\mathbf{P},\mathbf{Q},\mathbf{S}} \left\|\mathbf{D}\hat{\mathbf{X}} - \mathbf{P}\mathbf{Q}\right\|_F^2 + \lambda\|\mathbf{S}\|_1, \ \text{ s.t. } \mathbf{S} = \mathbf{PQ}.$$

For a fixed $\hat{\mathbf{P}}$ and $\hat{\mathbf{Q}}$, we update $\mathbf{X}$ by solving

$$\left(\mathbf{I} + \alpha\mathbf{D}^H\mathbf{D}\right)\mathbf{X} = \mathbf{Y} + \alpha\mathbf{D}^H\hat{\mathbf{P}}\hat{\mathbf{Q}}. \quad (3)$$

We iterate (2) and (3) are alternatively until convergence. $\mathbf{X}$ is initialized using the noisy images and $\mathbf{P}$ and $\mathbf{Q}$ are initialized by applying SVD to the noisy finite difference images. Note also that the low-rank model of $\mathbf{X} = \mathbf{UV}$ can readily be incorporated into the current formulation to further improve [11-15] its denoising performance.

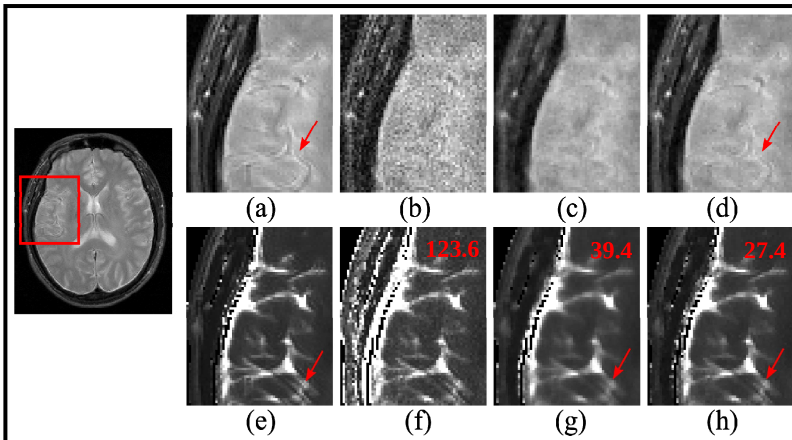

Fig. 2: Comparison of zoomed-in regions from T2-weighted images (a-d) and estimated T2 parameter maps (e-h), for (from left to right): gold-standard, noisy data, results from the method in [1] and the proposed method.

Results: We evaluated the proposed method using multiple sets of quantitative imaging data. One of them was a multi-echo image series acquired using a spin-echo sequence on a 3T Siemens Trio scanner with 12-channel head coil (32 echoes, 256x208 matrix size and 220x220mm^2 FOV). The original sum-of-squares images had very high SNR and were treated as the gold-standard. White Gaussian noise was added to simulate the noisy data. The proposed method was applied to denoise the images with $L = 4$, $\alpha = 2.0$, $\lambda = 0.005$ and five iterations between the subproblems in (2) and (3). The number of ALM iterations is 20. Fig. 2 compares the denoising results. The proposed method provides excellent noise reduction with better edge preservation, both in the contrast-weighted images and the parameter maps. The root mean square errors (RMSEs) for T2 estimation are also shown in the upper right corners of the T2 maps. Significant error reduction can be seen.

Conclusion: We have presented a new method to denoise image sequences for quantitative MR imaging. An efficient algorithm was described to solve the resulting optimization problem. Significant noise reduction with edge preservation has been demonstrated. The proposed regularization formalism can also be extended to joint reconstruction of image series by including proper linear operators that model the data acquisition process.

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