S-SPIRiT: An Iterative/Shrinkage Approach to SPIRiT for Real-Time Cardiac MRI

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Purpose: The high spatial and temporal requirements of real-time cardiac MR imaging under free-breathing conditions can be challenging to meet with current GRAPPA and SENSE reconstruction techniques. SPIRiT[1] improves upon GRAPPA by taking advantage of additional self-consistency constraints between acquired and unacquired data. Yet typical iterative solutions for SPIRiT with nonlinear regularization are susceptible to local minima issues due to ill-conditioning of the underlying optimization problem. The nonlinear conjugate gradient (NLCG) solver can be computationally intensive for SPIRiT with ℓ_1 regularization and is difficult to tune to specific scenarios. We propose shrinkage SPIRiT (S-SPIRiT), a FISTA-based[2] implementation of ℓ_1-regularized SPIRiT that provides improved robustness to suboptimal parameter tuning and reduced computational requirements compared to NLCG techniques.
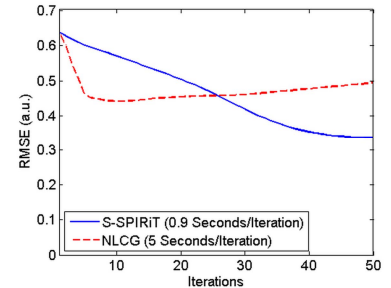

Figure 1: Phantom RMSE for S-SPIRiT and NLCG with additional noise. NLCG achieves optimal RMSE in 50 seconds, while S-SPIRiT surpasses NLCG in 25 seconds.
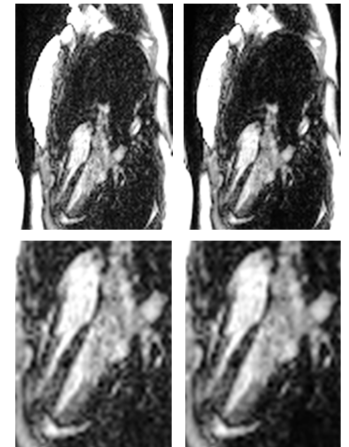
Methods: Given the cost function for unregularized SPIRiT, $g(x) = \|Fx - y\|_2^2 + \lambda\|Gx - x\|_2^2$, the ℓ_1-regularized formulation is then $f(x) = g(x) + \mu\|\Psi x\|_1^1$, with F, G, and Ψ denoting respectively the k-space sampling operator, the SPIRiT self-consistency operator and a sparse transform operator, x and y denoting respectively the reconstructed and acquired data, and λ and μ denoting tuning parameters. FISTA[2] minimizes $f(x)$ by first computing a gradient step $x - \nabla_x g(x)$ and then performing a shrinkage operation T_μ within the sparsifying domain: $\Psi^{-1}T_\mu\Psi(x - \nabla_x g(x))$. Fast $O(1/k^2)$ convergence is achieved by using a particular linear combination of the previous two iterations as the input to the next iteration. In S-SPIRiT, we apply FISTA to the standard SPIRiT cost function to achieve ℓ_1 regularization within the wavelet domain. The low cost of the gradient calculation for FISTA significantly reduces computational time compared to the multiple line searches required for the NLCG approach. In S-SPIRiT, we also use a dynamically updated wavelet thresholding parameter based on the ratio of the noise variance to subband signal as estimated from the wavelet coefficients[3], eliminating the need to tune an additional parameter and providing robustness in the presence of suboptimal parameter tuning and high noise levels.

S-SPIRiT was tested in both phantom and *in vivo* studies. In the phantom study, 80 frames of an axial slice of a static spherical water bottle phantom were acquired (Siemens, Avanto 1.5T, 12 channels) with no parallel acceleration. Data were retrospectively randomly downsampled in both spatial directions to rate 6. A 7x7 SPIRiT kernel was estimated using the temporal average of all frames. Both S-SPIRiT and NLCG reconstructions were performed in two scenarios: (1) without additional noise and (2) with additional complex Gaussian noise. For optimal performance, NLCG reconstruction utilized both wavelet domain and total variation (TV) regularization. Per iteration RMS error between results and the full k-space data was measured. Tuning parameters were optimized for the noiseless scenario and remained unchanged in the noisy scenarios. For the *in vivo* study, free-breathing cardiac cine data (uniformly downsampled, temporally-interleaved Cartesian trajectory, 32 channels) were acquired at rest (rate 6, 256 frames, 4-chamber view) and after treadmill exercise stress (rate 5, 50 frames, short axis view) at 1.5T (Siemens, Avanto) from one healthy volunteer. Data were first reconstructed using GRAPPA with a 4x5 kernel estimated from the temporal average of all frames. GRAPPA results were used to initialize both S-SPIRiT and NLCG reconstructions. A 7x7 SPIRiT kernel estimated using the temporal average of the initially acquired data was used. The minimum of the cost function was used as a stopping criterion for both reconstructions. For optimal performance, NLCG reconstruction used both wavelet and TV regularization terms. Average SNR based on random matrix theory was measured for both reconstruction methods. All reconstructions were performed using Matlab 2012a on an Intel Core i5 workstation with 16Gb memory.

Figure 2: In vivo NLCG (**left column**) and S-SPIRiT (**right column**) images. S-SPIRiT shows visibly reduced noise levels in both the lung region (**top row**) and within the myocardium and blood pool (**bottom row**).

Results: Figure 1 shows RMS error relative to the full k-space phantom data for both S-SPIRiT and NLCG the noisy scenario. The use of dynamic wavelet thresholding provides an additional degree of robustness, allowing S-SPIRiT to achieve reduced RMSE compared to NLCG in a shorter amount of time. Figure 2 shows a comparison between *in vivo* NLCG and S-SPIRiT results at rest during systole, where cardiac motion is most pronounced. At rest, S-SPIRiT produced a 14.80% SNR improvement over NLCG. Post exercise stress, S-SPIRiT produced a 33.44% SNR improvement over NLCG. For the phantom study, each NLCG iteration took 5 seconds, whereas each S-SPIRiT iteration took on average 0.9 seconds. In the *in vivo* study, NLCG required on average 16 seconds per iteration and typically converged within 10 iterations for a per frame reconstruction time of about 160s. S-SPIRiT required 3.5 seconds per iteration and converged within 25 iterations, or about half the reconstruction time of NLCG.

Conclusion: Compared to NLCG techniques, S-SPIRiT can provide an efficient implementation of ℓ_1-regularized SPIRiT with additional robustness towards suboptimal parameter tuning and high noise levels. S-SPIRiT may be a practical way to achieve improved image quality beyond GRAPPA in the context of free-breathing real-time cardiac MR imaging.

References: 1. Lustig M, Pauly JM. SPIRiT: Iterative self-consistent parallel imaging reconstruction from arbitrary k-space. Mag. Res. Med. 2010; 64(2): 457-71.
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