Motion residual reconstruction using low rank property of similarity patches in motion compensated compressed sensing dynamic MRI

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Introduction

Recently, many compressed sensing (CS) based algorithms have been developed for dynamic MR imaging applications by exploiting sparsity in temporal transform domain. For example, in k-t FOCUSS with motion estimation and compensation (ME/MC) [1], when a high resolution reference frame is available, ME/MC is shown a quite effective sparsifying transforms. However, one of the limitations of ME/MC is that the energy of the residual measurement after motion compensation is significantly reduced compared to the original k-space measurement. Hence, a new reconstruction algorithm for motion residual is required that judiciously reconstructs geometrically meaningful features. One of main contributions of this paper is a novel patch-based signal processing algorithm for motion residual reconstruction that overcomes the limitation of the existing k-t FOCUSS with ME/MC. More specifically, we impose a non-convex patch-based low-rank penalty that exploits self-similarities within the residual images. This penalty is shown to favor capturing geometric features such as edges rather than reconstructing the background noises. To solve the resulting non-convex optimization problem, we propose a globally convergent concave-convex procedure (CCCP)[2] using convex conjugate, which has closed form solution at each sub-iteration.

Theory

In imposing patch-similarities as a penalty, as shown in Figure 1, if aliased images are used for patch-based processing, the algorithm can be easily fell into a local minimum due to the incorrect search of similarity patches. The proposed algorithm provides a two-step hierarchical approach to address this issue. More specifically, we perform a novel patch-based non-local ME/MC using a diastole phase reconstruction as a reference, after which the patch-based low rank penalty is applied for motion residual image reconstruction. This is because a patch-based non-local ME/MC (NLMC) scheme removes the coherent aliasing and recovers most of the signal components, after which the patch-based low rank penalty effectively captures geometric similarities such as edges and boundaries from aliasing free residual image (Figure 1). The mathematical formulation is as following.

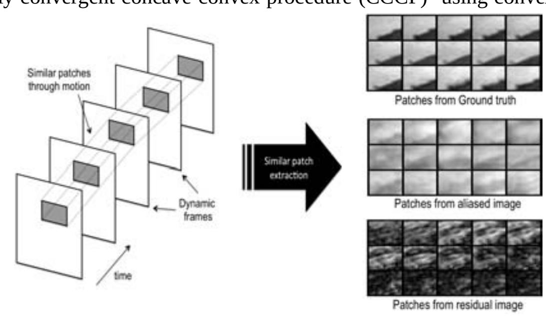

Figure 1. Comparison of similarity patches from ground-truth, aliased reconstruction and motion residual images. Patches from motion residual is less prone to aliasing artifacts.

Let cost function be represented as : $\mathrm{C(d)} = \frac{1}{2}||\mathrm{y} - \mathrm{F}\overline{\mathrm{m}} - \mathrm{Fd}||_F^2 + \psi(\mathrm{d})$, where prediction term $\overline{m}$ can be estimated by using NLMC algorithm[3]. Next step is to minimize the cost function C with respect to the residual signal d. The penalty term $\psi(\mathrm{d})$ assumes the following rank penalty: $\psi(\mathrm{d}) = \lambda \sum_p \mathrm{Rank}(V_p)$. Rather than using nuclear norm as a proxy for the rank penalty, we used the following non-convex rank prior $||V_p||_\nu = \sum_{k=1}^{\mathrm{Rank}(V_p)} h_{\mu,\nu}\left(\sigma_k(V_p)\right),\ 0 < \nu \leq 1$[4]. Here, the terms within summation represents a general Huber prior. Then, $\psi(d) = \lambda \sum_p ||V_p(\mathrm{d})||_\nu$ and the resulting cost function can be convexified as $\mathrm{C(W,d)} = \sum_{c=1}^{C} ||\mathrm{y_c} - \mathrm{F}\overline{\mathrm{m}}_\mathrm{c} - \mathrm{Fd_c}||^2 + \lambda \sum_p \min_\mathrm{W}(\frac{1}{\mu}||\mathrm{V_p(d)} - \mathrm{W_p}||_\mathrm{F}^2 + ||\mathrm{W_p}||_{\mathrm{g}_{\mu,\nu}})$ using a convex conjugate.[2] Hence, alternating minimization is guaranteed to converge. One main advantage of CCCP is that each sub-problem has a closed form solution. More specifically, W can be solved by simply solving singular value thresholding $\mathrm{W}_p^{(k+1)} = L\mathrm{shrink}_\nu(\Sigma,\mu)U^H$. Finally, the residual signal d can be calculated using conjugate gradient method.

Results

Figure 2 shows the reconstruction results of the radial data with acceleration factor 8. we have compared k-t FOCUSS, k-t FOCUSS with ME/MC, and the proposed algorithm. Note that k-t FOCUSS is prone to blurring near cardiac wall boundaries as also confirmed from temporal reconstruction profile (III,VI). For residual reconstruction, existing k-t FOCUSS with ME/MC (I) contained significant amount of background noises, whereas the proposed residual encoding (II) removes most of them and retains mainly geometrically meaningful features along cardiac wall boundaries. Due to the noise, the reconstruction results using k-t FOCUSS (III,VI) and k-t FOCUSS with ME/MC (IV,VII) reconstruction still contain the aliasing artifacts, where the results using the proposed method (V,VIII) are nearly aliasing free.

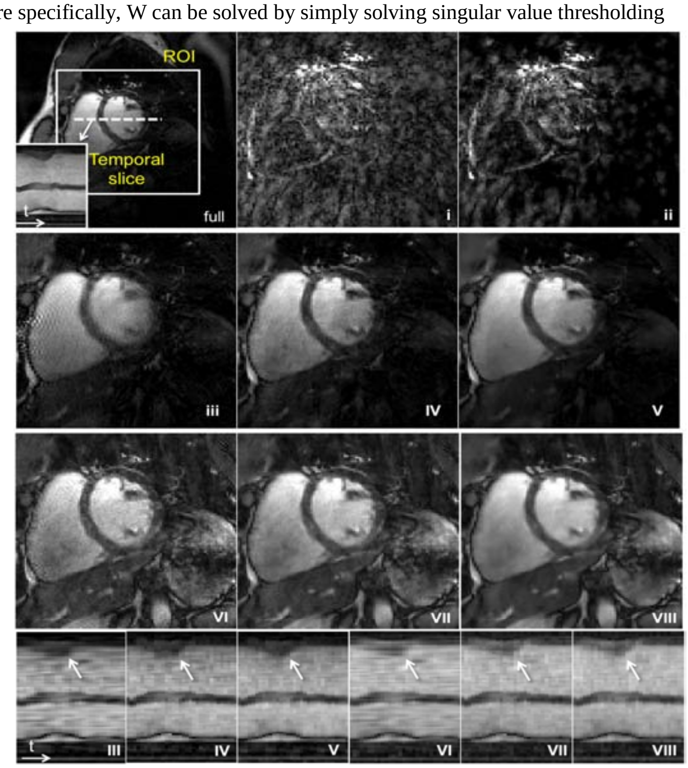

Figure 2. ROI and temporal slice images of the reconstruction image of radial data with acceleration factor 8. I,II: residual reconstruction of k-t FOCUSS with ME/MC and the proposed algorithm, respectively. III,IV,V: one coil image representing k-t FOCUSS, k-t FOCUSS with ME/MC, and the proposed algorithm, respectively. VI,VII,VIII: SOS image representing k-t FOCUSS, k-t FOCUSS with ME/MC, and the proposed algorithm, respectively.

Conclusion

In this paper, an improved motion compensated k-t FOCUSS algorithm using non-local motion compensation and patch-based residual encoding was proposed. In prediction step, a high resolution reference frame is generated during diastole phase, and NLMC improves the prediction accuracy. To judiciously use the residual signals during residual encoding, a residual encoding scheme using patch-based non-convex low-rank penalty was proposed. To deal with the non-convex and non-smooth penalty, a concave-convex procedure has been proposed. Extensive experimental results show that the proposed algorithm clearly reconstructs the important cardiac structures and outperforms existing k-t FOCUSS with ME/MC.

References

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