Blind compressive sensing dynamic MRI with sparse dictionaries

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Introduction:
The slow acquisition nature of MRI and the risk of peripheral nerve stimulation often restrict the achievable spatiotemporal resolution and volume coverage in several dynamic MRI applications. Several schemes that model the voxel time series as a sparse linear combination of basis functions in a fixed dictionary (eg: Fourier dictionary) were introduced to recover dynamic MRI data from undersampled measurements [1][2]. However, a challenge is the sensitivity of these methods to inter-frame motion, which decreases the sparsity of the representation; these methods suffer from temporal blurring at high accelerations. To address this, recently [3] proposed a blind compressive sensing (BCS) framework where the dictionary basis and the sparse coefficients were jointly estimated from undersampled data.
The BCS scheme demonstrated improved reconstructions of myocardial perfusion MRI data in comparison to existing methods such as compressed sensing and low rank methods [3]. However, we observe noise or alias patterns were learned by some of the basis functions in some methods at high accelerations; this resulted in speckle like residual artifacts in the reconstructed images (see fig.1). The main focus of this work is to further improve the BCS scheme by additionally constraining the dictionary. Specifically, we assume the basis functions to be sparse. Hence, we term the new method as sparse BCS.

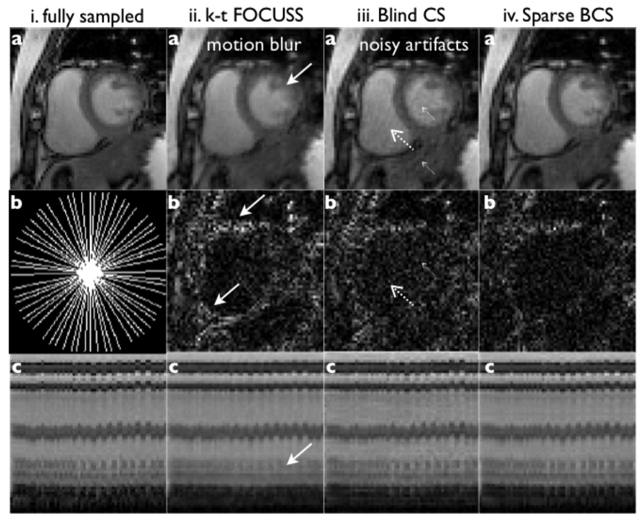

Fig.1 Comparisons using free breathing data: The first, second and third rows respectively correspond to a spatial frame, error image, and the image time profile. The sampling mask is shown in i.b. The error images are scaled up by ~5 fold for better visualization. The kt FOCUSS method resulted in motion blurring and loss of temporal resolution due to the large motion content (see arrows in ii). The BCS scheme maintained the motion content, but suffered largely from noisy artifacts due to learning of noisy bases (see arrows in iii.). In contrast, the sparse BCS scheme produced image quality with better spatiotemporal fidelity and reduced noisy patterns.

Methods: We model the dynamic signal Casorati matrix ($\mathbf{\Gamma}$) as the product of a spatial coefficient matrix and a dictionary matrix which contains the temporal basis functions. $\mathbf{\Gamma}_{MxN} = \mathbf{U}_{MxR}\mathbf{V}_{RxN}$; Here R is the number of basis functions in the dictionary, M and N are respectively the number of voxels in each frame and the number of time frames. We impose sparsity constraints on **U** and **V** and simultaneously estimate them by solving:

$$\{\mathbf{U}^*,\mathbf{V}^*\} = \arg\min_{\mathbf{U},\mathbf{V}} \|A(\mathbf{UV})-\mathbf{b}\|_2^2 + \lambda_1\|(\mathbf{U})\|_1 + \lambda_2\|(\mathbf{V})\|_1 ; \qquad (1)$$

Here A is the Fourier sampling operator that acquires the measurements **b** on a specified k-t trajectory. The first term ensures data consistency while the regularizing terms make the problem well posed. The main difference with the BCS setting is the sparsity constraint on **V** instead of the Frobenius norm constraint on **V** that we assumed in [3]. The experimental results show that this constraint effectively penalizes the basis functions that capture noisy oscillations, which corrupt the BCS basis functions. The optimization problem in (1) is non convex; it is convex with respect to one variable, if the other is assumed to fixed. Similar to [3], we solve it by using a majorize minimize algorithm, which involves the following minimization scheme:

$$\{\mathbf{U}^*,\mathbf{V}^*\} = \arg\min_{\mathbf{U},\mathbf{V},\mathbf{L},\mathbf{R}} \|A(\mathbf{UV})-\mathbf{b}\|_2^2 + \lambda_1\|(\mathbf{L})\|_1 + \lambda_2\|\mathbf{R}\|_1 + \beta_1\|(\mathbf{U}-\mathbf{L})\|_2^2 + \beta_2\|(\mathbf{V}-\mathbf{R})\|_2^2 ;(2)$$

Here β_1 and β_2 are the continuation parameters that determine the accuracy of the algorithm. (2) converges to (1) when β_1, β_2 tends to infinity . For a fixed $\beta 1$ and $\beta 2$, the algorithm iterates between simple steps of (a) shrinkage of **U**, to update **L**, (b) shrinkage of **V**, to update **R**, (c) quadratic problem in **U** (solved by using a CG gradient algorithm), and (d) quadratic problem in **V** (solved by CG). To minimize the risk of convergence to local minima, we use a continuation strategy. Specifically, we iterate the above four steps, starting with low values $\beta 1$ and $\beta 2$. The problem in (2) reduces to the low rank solution for low values of continuation parameters. We gradually increment these parameters, thus attenuating the low coefficients and basis functions. This continuation scheme is seen to provide good convergence.

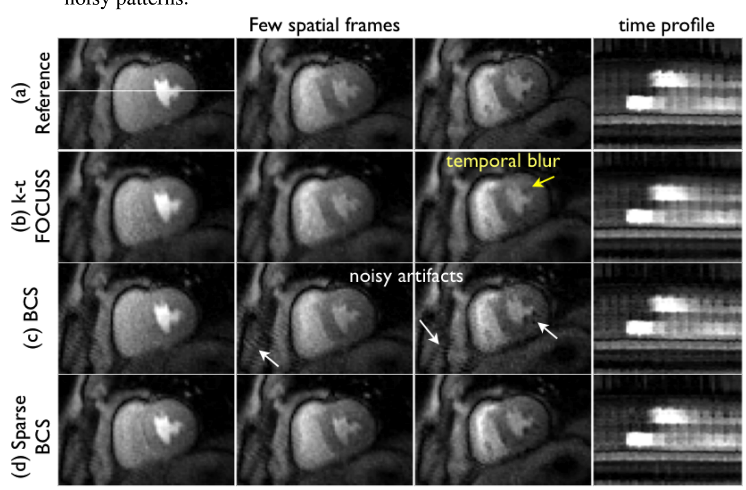

Fig.2 Comparisons using free breathing stress myocardial perfusion MRI data: Each row shows few spatial frames and the image time profile. We observe a similar behavior as seen in fig.1. k-t FOCUSS showed some temporal blur (see yellow arrow in (b)). BCS had better temporal fidelity but suffered from noisy artifacts (see arrows in (c) due to learning noisy patterns. Sparse BCS resulted in reconstructions with reduced noise like artifacts without compromising on the spatiotemporal fidelity.

Results: To evaluate the proposed scheme, we perform retrospective undersampling experiments on fully sampled reference data. We compare the proposed sparse BCS scheme with the BCS scheme [3] and the k-t FOCUSS scheme [5] that utilizes Fourier dictionaries. In figure 1, we consider data from a free breathing cardiac scan (N_{PE} X N_{FE} x time: 128 x 128 x60; time resolution ~ 1sec). The data had significant inter-frame motion (see ripples in the time profile of fig 1). A radial trajectory with 30 radial rays with golden ratio angle spacing between rays was used for undersampling. In fig 1, we observe k-t FOCUSS to exhibit motion blurring and temporal smoothing due to large motion content. The BCS scheme was robust to these compromises and preserved the motion well. However, it suffered from noisy artifacts due to few of the bases capturing alias patterns. In contrast, the sparse BCS scheme penalized these noisy patterns and provides superior reconstructions.
In figure 2, we show a second example on free breathing cardiac perfusion data. This data was acquired using a radial FLASH saturation recovery sequence (TR/TE = 2.5/1.3 ms; 5 slices, 72 radial rays uniformly spaced in each frame with uniform rotations across frames, 256 read out points, 4 coils). We considered reconstructing a subset of this data. Specifically we performed a single coil single slice reconstruction using 24 radial rays. We observe similar trends as in fig.1. Specifically kt FOCUSS had temporal blurring in some frames, BCS learned noisy bases and had noisy artifacts. Sparse BCS was robust to these compromises and provided reconstructions with superior spatio-temporal fidelity.

Discussion: We proposed an algorithm to learn dictionary atoms that are constrained to be sparse from undersampled kspace data for dynamic MRI reconstruction. Our experiments demonstrate that by promoting sparsity on the dictionary atoms, the learning of noisy basis functions can be considerably reduced.

References: [1] Lustig et al MRM 2008, [2] Otazo et al MRM 2010, [3] Lingala et al ISMRM 2012, [4] Recht et al SIAM Review 2010, [5] Jung etal MRM 09