Unifying Compressed-Sensing Reconstruction Framework for Multidimensional MRI: Combining Novel Dictionary Models with Frame-Based Sparsity and Flexible Undersampling Schemes

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Purpose: Multidimensional magnetic resonance imaging (MRI), e.g. dynamic cardiac perfusion MRI or high angular resolution diffusion imaging (HARDI) for investigating brain's neural structure, is typically time consuming with the associated applications strongly motivating faster acquisition schemes by exploiting principles in compressed sensing [1]. We propose a novel *unified framework for reconstructing multidimensional MR images from undersampled k-space* acquisitions. While leading compressed-sensing reconstruction methods employ either L_1 analysis or synthesis approaches using mathematical frames (e.g. overcomplete wavelets), approaches using dictionaries ignore the frame-based L_1 sparsity constraints. The proposed framework incorporates a novel method *combining frame-based L_1 analysis with dictionary-based sparsity* (related to L_1 synthesis). While we propose a *low-rank spatiotemporal dictionary* model for dynamic MRI, we propose a *concise rotation-invariant dictionary* for HARDI. We employ overcomplete wavelet frames to enforce sparsity as well as *multiscale regularity* in the spatial domain. Results on simulated and clinical multidimensional MRI demonstrate improved results using the proposed framework.

Methods: Our contributions are in (i) dictionary modeling and (ii) formulating the reconstruction problem for multidimensional MRI.

<u>(I) Dictionary Modeling:</u> For dynamic MRI, we propose a *spatiotemporal dictionary model* where each atom is a spatiotemporal patch of intensities that captures the joint coherence in space and time for cardiac perfusion MRI. We have proposed a method for learning spatiotemporal dictionaries by enforcing a low-rank constraint during the optimization. For HARDI, we propose *rotation-invariant dictionaries* that enable a concise dictionary (few atoms representing key diffusion-signal types) by explicitly optimizing the rotation for each atom during sparse fitting.

<u>(II) Multidimensional MRI Reconstruction:</u> Given (i) undersampled multidimensional MRI z (complex) associated with N timepoints (dynamic MRI) or N gradient directions (HARDI), and (ii) a dictionary with atoms $\{d_i\}$, we define the reconstructed multidimensional image u^{opt} (complex) as:

$$\operatorname*{argmin}_{u} \min_{R,c,P} \left[\lambda \log(\|\psi u\|_1 + \varepsilon) + (1-\lambda) \sum_j \left\| u_j - P_j \otimes \sum_i c_{ji} R_{ji}(d_i) \right\|_2^2 \right] \text{ such that } \|\Im u - z\|_2^2 \le \eta; \forall j, \|c_j\|_0 \le \tau ,$$

where $\lambda \in [0,1]$ is a free parameter, ψ is a tight-frame analysis transform (we use an overcomplete wavelet transform applied separately to each spatial image), ε is a tiny positive constant, $u_j \in C^{S*N}$ comprises SxN complex signal values along a multidimensional patch around voxel j with S voxels in the spatial dimension for each of N voxels along the non-spatial dimension (for dynamic MRI u_j is a spatiotemporal patch of size 4xN; for HARDI u_j is just a vector of size 1xN along the gradient-direction dimension), P_j is a SxN multidimensional patch where each component is a unit-magnitude complex number capturing phase, $\otimes$ denotes element-by-element multiplication of two matrices or vectors, $c_j \in \Re^I$ comprises coefficients $c_{ji} \in \Re$ associated with each atom i that best fits u_j, $\Im$ is the undersampled Fourier transform, η is the noise variance, and $R_{ji}(\cdot)$ is an operator that (i) is fixed to identity in case of dynamic MRI and (ii) denotes a rotation of atoms, in case of HARDI, where $R_{ji}(d_i)$ is the rotated i[th] atom that best fits u_j.

Results: The following two figures show the fully-sampled data, example atoms in the dictionary employed, the simulated undersampling patterns, and subsequent reconstructions. Our experiments have shown that $\lambda \in [0.3,0.7]$ typically gives the most desirable results.

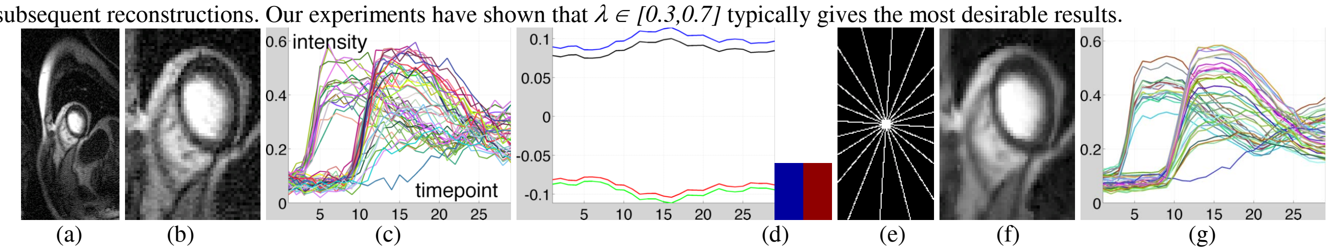

Figure 1: Reconstruction of dynamic cardiac perfusion MRI from highly-undersampled k-space (R=11.5). (a) Fully-sampled data (192x96 voxels in space; 29 timepoints), 1 timepoint shown. **(b)** Zooming into the heart region. **(c)** Time curves for pixels in the heart region. **(d)** A 2x2 spatiotemporal dictionary atom with 4 temporal curves (left) and the mean of those curves capturing the spatial pattern (right). **(e)** k-space undersampling scheme. **(f)** Reconstructed image. **(g)** Temporal curves for the reconstructed image.

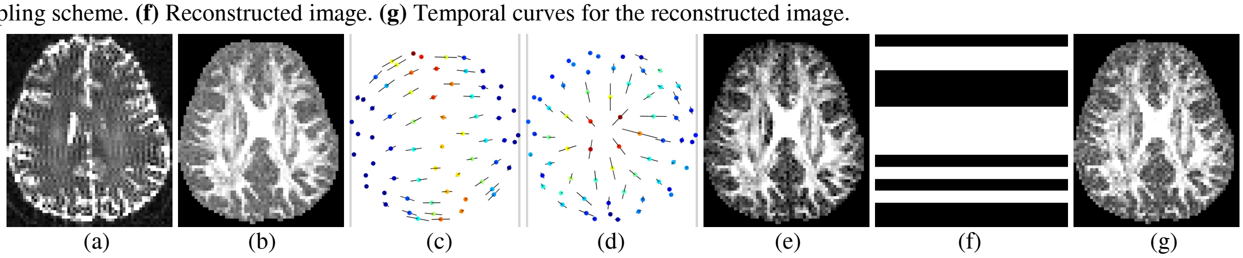

Figure 2: Reconstruction of HARDI images from undersampled k-space in addition to subsampled gradient directions. (a) B0 image; 108x108x40 voxels; 64 gradient directions. **(b)** GFA image from noisy fully-sampled data. **(c)-(d)** Example atoms in concise rotation-invariant dictionary representing a voxel with single tract and a voxel with two crossing tracts, respectively. **(e)** GFA map from dictionary-fit to data. **(f)** k-space undersampling pattern; can be used with time-multiplexed EPI MRI. **(g)** Reconstructed GFA image using 1.5X subsampled directions and 1.33X undersampled k-space for the chosen directions.

Discussion and conclusions: We have presented a flexible unified framework for reconstruction of multidimensional MRI from undersampled data. The framework is flexible with regard to the kinds of imaging or undersampling strategies that can be exploited in combination with compressed-sensing principles. Similarly, the framework is flexible in the kinds of sparse models that can be enforced on the data, allowing a variety of wavelet models, total-variation models, as well as dictionary models. Future work involves validation for clinical use and extension to multi-shell imaging.

References: [1] Awate et al. IEEE Symposium on Biomedical Imaging. 2012, pp 318-321. IEEE Xplore.