Improving Compressed Sensing Initialization and Convergence Using an Efficient Auto-Calibrating Parallel Imaging Method

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INTRODUCTION: Recently, compressed sensing (CS) and parallel imaging (PI) have been successfully combined to exploit both signal sparsity and coil sensitivity for accelerated MRI. Among such approaches, L₁SPIRiT [1], which synergistically integrates CS and PI, has demonstrated promising results in clinical evaluations. An eigenvector-based L₁SPIRiT approach (ESPIRiT) [2] was developed to reduce the computation complexity of original L₁SPIRiT. However, L₁SPIRiT and ESPIRiT still require a non-linear iterative solver and a large number of iterations for convergence resulting in highly intensive computation. Previous work has demonstrated the feasibility of faster CS convergence by improving the initial condition using auto-calibrating PI (ac-PI) [2,3].

However, such methods necessitate k-space sampling with reduced randomness, which is undesirable for CS. Furthermore, such modified sampling patterns still generate over a thousand synthesis patterns requiring considerable processing time for ac-PI initialization. This work investigated an efficient ac-PI initialization method for faster CS reconstruction without modification of the sampling pattern and demonstrated its feasibility using ESPIRiT.

METHODS & MATERIALS: ac-PI processing time increases in proportion to the number of synthesis patterns (N_{pat}) in calibration and the square of the number of coil channels (N_{coil}) in synthesis. To address the random k-space sampling needed for CS and large N_{coil} for high acceleration, our proposed method adapts and combines a data decoupling calibration method for non-uniform Cartesian sampling [4] and direct coil combined synthesis [5] to generate an improved initial solution with efficient computation.

As shown in Fig. 1, from central k-space calibration data (Fig. 1a), we first estimate coil sensitivity (Fig. 1b) based on Eigenvector computations for ESPIRiT reconstruction and also generate low-resolution coil-by-coil (CBC) images (Fig. 1c) by Fourier transform. The CBC images were coil-combined (Fig. 1d) and then converted to k-space to obtain coil combined (CC) calibration data (Fig.1 e). Next, we perform calibration by fitting the CBC calibration data to the CC calibration data and directly reconstruct k-space in a single CC channel.

Calibration: Conventional calibration uses pseudo-inversion to compute optimal coil weights (*W*) for each synthesis pattern separately. On CS datasets with $N_{pat} > 10^4$, the dominant computation is the matrix inversion in the pseudo-inversion process and this matrix inversion requires $O(N_s^a)$ multiplications (N_s : number of source data in a pattern, σ : typically ~3). Similar to [4], we decouple the N_s source data into: N_{s1} k_x and coil sources for each k-space source line and N_{s2} source lines with different k_y-k_x shifts in a synthesis pattern. *W*'s are calculated in the following two steps. Step 1: find the optimal weights (W_1) to fit k_x and coil sources and W_1 for each potential k_y-k_x shift is to CC k-space target. Then, calculate a synthetic calibration dataset using k_x and coil sources and W_1 for each potential k_y-k_x shift; Step 2: for each synthesis pattern, find the optimal weights (W_2) to combine step-1 syntheses from different k_y-k_x shifts based on min $||\cdot||^2$ fitting on synthetic calibration data from step 1. Finally, W_1 and W_2 are combined as the final coil weights ($W=W_1 \times W_2$) for data synthesis. As shown in Fig. 2, this method resolves pattern-by-pattern computation on large matrixes in conventional calibration (a) to computation on smaller matrixes for only N_{line} (N_{line} <50) k_y-k_x shifts in step 1 (b, upper) and computation on very small matrixes ($N_{s2} \approx 6$) over different patterns in step 2 (b, lower).

Synthesis: each datum in the coil-combined k-space is synthesized using sampled sources from the original k-space data with the pre-determined *W*'s. Such direct synthesis in CC k-space reduces the computation complexity of conventional CBC synthesis by N_{coll} . Similar to ARC [6,7], for faster computation, k-space data were transformed into hybrid space with Fourier transform along k_x and data synthesis was performed in hybrid x-k_y-k_z domain in this work.

Generate initial solution for CS: Finally, the coil sensitivity is reapplied on the above CC solution to generate CBC k-space data as the initial solution for subsequent ESPIRiT reconstruction.

In this study, the conventional and the proposed efficient ac-PI methods were implemented in C++ and are referred to as ARC and e-ARC. To evaluate the proposed method, we collected 4 full kspace datasets (2 proton-density-weighted knee & 2 T₂-weighted brain) from healthy volunteers on GE 1.5\3T scanners using 8-ch phased array coils. Full k-space datasets were subsampled in offline to simulate 2.5×2.5 acceleration with Poisson disk sampling [1] and then processed using ESPIRiT with different initialization (1. conventional zero-filling (ZF); 2. ARC & 3. e-ARC). Different reconstructions were compared based on reconstruction accuracy and processing time. **RESULTS:** Fig. 3 shows the convergence of ESPIRiT with different initialization on a knee dataset. Both ARC and e-ARC significantly improved the initial solution and as a result ESPIRiT converges much faster than ZF initialization. Fig. 4 compares different initial solutions and ESPIRiT reconstructions on a brain dataset. ARC and e-ARC generate much more accurate reconstruction than ZF. Using ARC and e-ARC initialization, ESPIRiT in 15 iterations provides image quality similar to ESPIRiT with ZF after 45 iterations. On all the datasets evaluated in this study, the mean error for the initial solution is 31.57% for ZF. 17.98% for ARC and 19.08% for e-ARC. The mean reconstruction error is 17.38% for ESPIRiT with ZF initialization and 45 iterations and 15.77% and 16.28% for ESPIRiT with ARC and e-ARC initialization and 15 iterations, respectively. The average processing time is \sim 130 sec in calibration and \sim 20 sec in synthesis for ARC, and is reduced to \sim 2 sec in calibration and ~5 sec in synthesis for e-ARC with single-thread computation.

DISCUSSION: This work demonstrated a method for highly efficient ac-PI reconstruction from random Cartesian k-space sampling. Our preliminary experiments show that this method could provide slightly higher ESPIRiT reconstruction accuracy within 3× less iterations compared to conventional ESPIRiT with ZF. The proposed method is promising for improving the initial condition and therefore significantly increasing the convergence speed of CS reconstruction.

References: 1. Lustig ISMRM, 2009:334; 2. Lai, ISMRM, 2010:345; 3. Lai, ISMRM, 2011:68; 4. ; 4. Lai, ISMRM 2012:4245; 5. Beatty, ISMRM 2007:1751; 6. Beatty, ISMRM 2007:1749; 7. Brau, MRM 2008:382







Fig. 2. Conventional (left) vs. data decoupling (right) calibration





Fig. 4. Initial solution (upper) and converged solution (lower) for zero-filling (a,b), ARC (c,d) and e-ARC (e,f)