Fast Compressed Sensing Reconstruction using a Direct Fourier-Wavelet Transform

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Introduction: High computational complexity is one of the major issues for compressed sensing (CS) reconstruction. Fast iterative thresholding methods [1,2] have been extensively studied as alternatives to convex optimization for large-sized problems [3], but the main computational bottleneck still exists due to serial application of Fourier and wavelet transforms. In this work, we present a new way to reduce the computational complexity for the CS reconstruction by directly transforming between k-space and wavelet domains. We demonstrate the reduced reconstruction time using 4D breast DCE-MRI data, of a size that is often challenging for CS reconstruction.

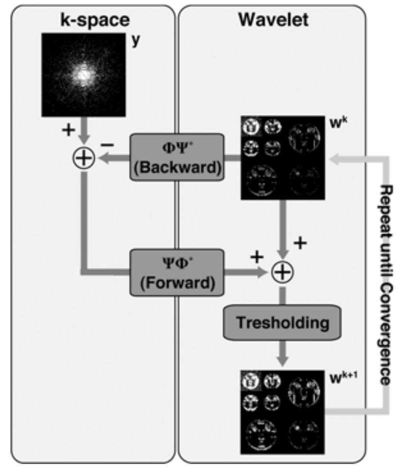

Fig 1: A diagram of iterative thresholding methods. The main computational load is the matrix-vector operations ($\Phi\Psi^*$ and $\Psi\Phi^*$).

Methods: CS allows reconstructing an image by solving the following convex optimization:

$$\text{Minimize } \|w\|_1, \quad \text{s.t. } \| y - \Phi\Psi^* w \|_2 < \varepsilon,$$

where Φ is the undersampled Fourier transform, y is acquired k-space samples, Ψ is a wavelet transform, w is wavelet representation, and ε bounds the amount of noise in the measurement. One way to solve this minimization is to use fast iterative thresholding methods, as shown in Fig 1. Iterative thresholding reconstruction primarily includes fast matrix-vector multiplications (forward $\Psi\Phi^*$ and backward $\Phi\Psi^*$ processes) and thresholding. The core computational load is the forward (from k-space to wavelet) and backward (from wavelet to k-space) processes.

Fig 2a illustrates a conventional way to implement the forward process (the inverse Fourier transform Φ^* followed by the wavelet transform Ψ). We propose to directly apply the wavelet transform operation (low- and high-pass filtering, and downsampling) in k-space (see Fig 2b) to avoid finite impulse response (FIR) filtering. This can be more computationally efficient since the FIR filtering in the image domain is replaced with a multiplication in k-space. The spectral weightings D_H and D_L are known based on the wavelet filters. Note that the two approaches (Fig 2a and b) are not exactly identical since the FIR filtering can have different boundary adjustments, but the difference is extremely subtle.

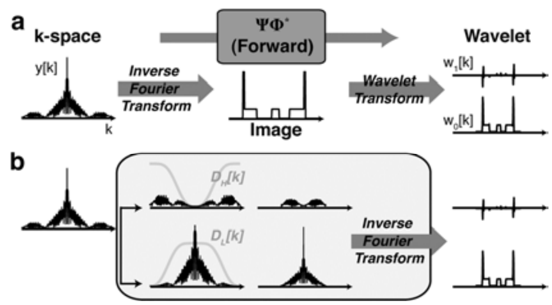

Fig 2: Illustration of the forward process, consisting of the inverse Fourier transform Φ^* followed by the wavelet transform Ψ.

All reconstructions were implemented on a Linux PC equipped with a dual six-core 2.66 GHz CPU (Intel Xeon) and 64 GB of memory using MATLAB R2011b. We first evaluated the actual computation time on a single set of core operations (forward and backward processes) with 100 repetitions. For a 2D image (512×512), we used the Daubechies wavelet filters and disabled the Matlab Executable (MEX)-function for the FIR filtering to accentuate the computation time difference. For 3D images, we used the Farras filters [4] and enabled the MEX-function for the FIR filtering (upfirdn.m).

Secondly, we employed the proposed method into an approximate message passing (AMP) algorithm [5], a variation of iterative soft-thresholding, and measured the actual reconstruction time with 100 repetitions. Breast DCE-MRI data were retrospectively undersampled by a factor of 4.1, and the AMP reconstruction was applied on each 3D volume (244×128×48). We used the 3D Fourier and 3D wavelet transforms (enabled the MEX-function), and fixed the number of iterations to be 20 for all cases.

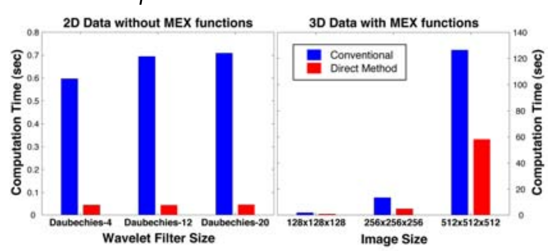

Fig 3: Average computation time on the forward and backward processes for 2D and 3D data.

Results: Fig 3 shows the average computation time on one set of the core operations (forward and backward processes). For 2D data, the computation time using the conventional approach increases as the wavelet filter size increases, while the computation time using the proposed method remains the same. More importantly, the average computation time (0.04 sec) is only 6 – 7% of the conventional approach (0.6 – 0.7 sec). For 3D data, the computation time using the proposed method is 60% faster than the conventional approach, even with the efficient MEX-function 3D wavelet transform.

Fig 4 shows the mean and standard deviation of CS reconstruction time based on AMP for all 20 temporal frames. The reconstruction time using the proposed method (12 sec) is reduced by 46%, compared with the conventional way (23 sec), while both reconstruction methods maintain good image quality with the same reconstruction error (see Fig 4).

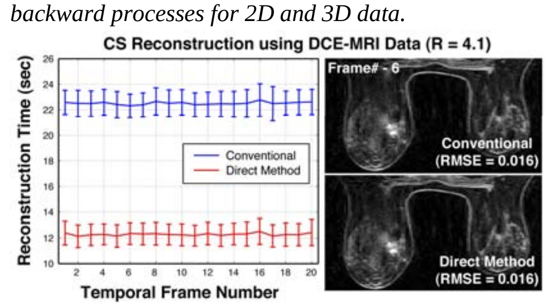

Fig 4: CS reconstruction time (mean and standard deviation) based on AMP using the conventional and direct methods.

Conclusion: We proposed a direct Fourier-Wavelet transform that can directly map between k-space and wavelet domains. The method replaces FIR filtering in the image domain with a multiplication in k-space, which can reduce computational complexity. Based on the AMP algorithm, we have shown that the actual CS reconstruction time can be reduced by 46% on MATLAB. This efficient computation can benefit almost all CS methods that exploit wavelet sparsity, and we anticipate further improvements will result from the code implementation in C.

References: [1] Donoho, IEEE TIT 1995, [2] Daubechies et al., Comm Pure Appl. Math 2004, [3] Donoho, IEEE TIT, 2006, [4] Abdelnour et al., IEEE ICASSP, 2001, [5] Donoho et al., PNAS, 2009