Iterative Hard Thresholding and Matrix Shrinkage (IHT+MS) for Low-Rank Recovery of k-t Undersampled MRI Data

Mark Chiew[1], Karla L Miller[1], Peter J Koopmans[2], Elizabeth M Tunnicliffe[3], Stephen M Smith[1], and Thomas Blumensath[4]

[1]*FMRIB Centre, University of Oxford, Oxford, United Kingdom,* [2]*Donders Institute for Brain, Cognition and Behaviour, Radboud University Nijmegen, Nijmegen, Netherlands,* [3]*AVIC, Nuffield Department of Clinical Medicine, University of Oxford, Oxford, United Kingdom,* [4]*ISVR, University of Southampton, Southampton, Hampshire, United Kingdom*

Purpose: In recent years, a technique has emerged for the recovery of under-sampled matrix data, called matrix completion[1]. In contrast to compressed sensing, the data are not required to be sparse - rather, the matrix needs to be of low rank for successful recovery to occur. Further, these methods do not require a specified basis set (as is used in compressed sensing) but effectively estimate a basis that gives the best low-rank approximation of the data. These methods start from the observation that an $m \times n$ matrix of rank r has $r(m+n-r)$ free parameters, compared to mn for a full rank matrix. Hence, it is possible to recover a low-rank matrix or approximation from a dataset that is not fully sampled. This concept has recently been applied to reconstruction of undersampled k-t MRI data (cardiac cine[2] and dynamic contrast enhanced[3]), where small amounts of coherent motion or signal enhancement in a static background produce data suitable for low-rank matrix recovery. Here we describe a novel recovery algorithm designed for rank-reduced approximation of undersampled MRI data, based on iterative hard thresholding[4] (IHT). Called IHT+MS (matrix shrinkage), we evaluate its ability to recover both undersampled k-t fMRI and cardiac data compared with the iterative rank power factorization (IRPF)[2] (cardiac only) and rank-constrained fixed point continuation approximation (FPCAr)[5] methods.

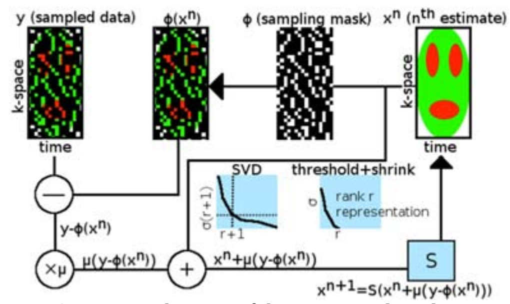

Figure 1 – Schematic of the IHT+MS algorithm.

Methods: The IHT+MS algorithm can be summarised (see Fig. 1):

$$x^{n+1} = S\left(x^n + \mu\left(y - \Phi(x^n)\right)\right)$$

where x^n represents the n^{th} iteration of the estimated k-t matrix, y is the sampled data, Φ is the sampling operator that selects measured matrix entries, and μ is a step size parameter. The shrinkage operator S uses an SVD to find the r+1th singular value σ_{r+1}, after which all singular values are shrunk to $\max(\sigma - \sigma_{r+1}, 0)$. Only the first r singular values survive this shrinkage and thresholding, producing a rank r data estimate. Because real MRI data is only ever approximately low rank, algorithm recovery performance can depend on the decay rate of singular values. While IHT performs well where singular values decay quickly, the IHT+MS algorithm improves performance in slow decay regimes. All tested algorithms were implemented in MATLAB. Resting state fMRI data were acquired at 3T (106x106x32 spatial points, 512 time points at TR = 836 ms). Retrospective sampling was performed on k_z by fully sampling 4 centre planes and undersampling the remaining 28 planes by 7x for an effective undersampling factor of 4x. Both algorithms targeted rank 128 reconstructions. Short axis cardiac cine data were acquired at 3T (interpolated 352x512 spatial points, 100 phases in a single cardiac cycle). Sampling used 8 centre k_y lines and 4/8/16x random undersampling factors on the other 504 lines, and were reconstructed at rank 20. Sliding window data were regularly undersampled to the same factor.

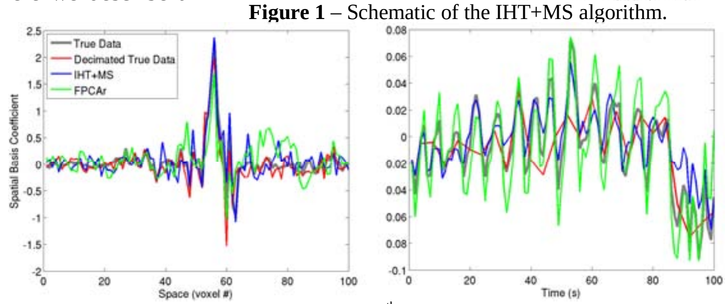

Figure 2 – Comparison of a portion of 4th fMRI principal components in the true data, data decimated to match undersampling, IHT+MS and FPCAr data. Results from first 3 principal components were visually indistinguishable.

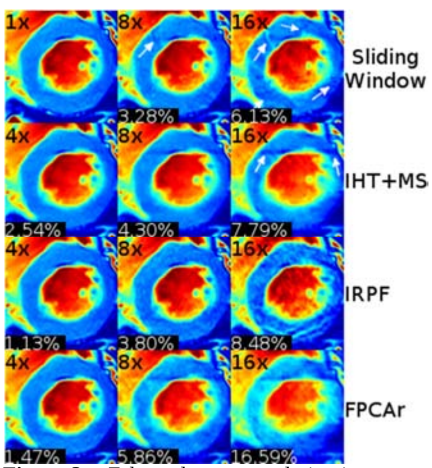

Figure 3 – False colour zoomed cine images at peak systole for 4/8/16x undersampling.

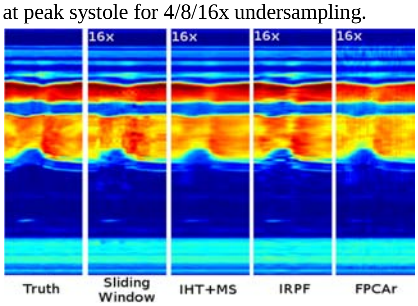

Figure 4 – Time courses of a line through the cardiac images at 16x undersampling.

Results: In the fMRI data, IHT+MS produced the best estimates of the first 100 fMRI PCA components (Fig. 2, see companion abstract for more details). Cardiac reconstruction fidelity was assessed using the relative Frobenius norm error between the true and reconstructed magnitude x-t data and visual inspection of reconstructed images (Figs. 3,4). In the 4x undersampled cardiac data, the IRPF method produced the lowest error, compared to 1.9% for sliding window (image not shown). At 8x and 16x, all images show visible artefacts, although qualitatively the IHT+MS images look least affected despite slightly higher error values than the sliding window method. In Fig. 3, artefacts are least apparent in the IHT+MS images. The IHT+MS images instead appear smoothed or filtered, suggesting a graceful degradation of the spatio-temporal point spread function from the reconstruction algorithm. Approximate reconstruction times for the cardiac data were 150, 90 and 180 min respectively for the IHT+MS, FPCAr and IRPF methods (16x4 core 2.66 GHz, 64 Gb RAM).

Discussion: The IHT+MS algorithm shows excellent ability to recover a low rank approximation of undersampled MRI data, even at undersampling factors beyond the degrees of freedom sampling limit. Qualitatively, both the spatial and temporal data reconstructed with IHT+MS show the least amount of artefact contamination, although error values can appear high because the algorithm does not preserve overall signal power. This may be irrelevant for applications such as resting state fMRI, in which temporal correlations are more important than signal amplitudes. Convergence is reasonably fast, and efficient SVD approximation methods can be used, particularly when the k-t matrix is highly non-square. Such rank-constrained reconstruction algorithms can be sensitive to the choice of rank, and prior information can be useful in selection of an optimal rank. Finally, these results do not consider multiple coils, and we expect future integration of IHT+MS with multi-coil measurements to produce higher fidelity reconstructions.

References: 1. Candes EJ et al. Found Comp Math. 2009;9(6):717–77 2. Haldar JP et al., IEEE-ISBI;1052–1055. 3.Lingala S et al. IEEE-TMI 2011;30(5):1042–1054. 4. Blumensath T et al. Appl and Comp Harm Anal. 2009;27(3):265–274. 6. Goldfarb D et al. Found Comp Math. 2011;11(2):183–210.