VOI-based Fourier Transform method for rapid partial calculation of B_0 maps from sparse susceptibility distributions

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Introduction: A method for calculating the B_0 field map resulting from an arbitrary magnetic susceptibility distribution via Fourier Transform-based convolution of the susceptibility distribution with a dipole kernel is well-known [1, 2]. The method involves a forward and inverse Fourier Transform of the entire susceptibility matrix, which is an operation that can have high computational memory and time requirements for large matrices. Recently, a more efficient method for calculating the resultant B_0 field maps in individual slices of a subject was developed [3]. However, this method is still time-inefficient when calculating the B_0 maps of multiple slices. Here, we present a VOI-based (volume of interest-based) approach that could reduce calculation time as compared to the traditional method or the slice-based method. The new method has generally lower memory requirements than the traditional method and can be faster than the traditional or slice-based methods when used on "sparse" matrices wherein the susceptibility sources and regions of interest comprise a relatively small portion of the total matrix. The VOI-based method can therefore be expected to accelerate certain applications, such as passive shimming simulations and patient-specific, localized B_0 field inhomogeneity calculations derived from anatomic scans.

Methods: The original Fourier Transform method [1] uses a convolution of a dipole Green's function kernel with the susceptibility distribution to calculate the resultant B_0 inhomogeneity map. In this method, the Green's function is computed in the *k*-space domain.

$$\Delta B(\vec{r}) = B_0 \cdot FT^{-1}\left\{G(\vec{k}) \cdot FT\{\chi(\vec{r})\}\right\}, \text{ where } G(\vec{k}) = \frac{1}{3} - \frac{k_z^2}{k_x^2+k_y^2+k_z^2}$$

The method published by Cheng et al. [2] instead computes the Green's function in the image domain, noting that this results in proper discretization of the Green's function in *k*-space.

$$G(\vec{k}) = FT\left\{\frac{1}{4\pi} \cdot \frac{2z^2-x^2-y^2}{(x^2+y^2+z^2)^{\frac{5}{2}}}\right\}$$

This can be taken a step further by noting that this equation can be used to express the effect of a single source susceptibility voxel at $\vec{r}'$ on the ΔB_0 field in a single target voxel at $\vec{r}$, wholly in the image domain.

$$\Delta B(\vec{r},\vec{r}') = B_0 \cdot G(\vec{r}-\vec{r}') \cdot \chi(\vec{r}')$$

$$\Delta B(\vec{r},\vec{r}') = B_0 \cdot \frac{1}{4\pi} \cdot \frac{2(z-z')^2-(x-x')^2-(y-y')^2}{((x-x')^2+(y-y')^2+(z-z')^2)^{\frac{5}{2}}} \cdot \chi(\vec{r}')$$

By integrating across $\vec{r}$ and $\vec{r}'$, this approach can be used to calculate the effect of a block of source susceptibility voxels on the ΔB_0 field in a block of target voxels comprising the VOI, even if the source and target blocks are not coincident and occupying the entire field of view (FOV) as is the case in the existing calculation method. The proposed new method is therefore a generalization of the existing calculation method. By applying the convolution theorem and utilizing the Fourier Transform for improved calculation speed, the following expression for the new VOI-based calculation method is obtained.

$$\Delta B(\vec{r},\vec{r}') = B_0 \cdot FT^{-1}\{FT\{G(\vec{r}-\vec{r}')\} \cdot FT\{\chi(\vec{r}')\}\}, \text{ where } G(\vec{r}) = \frac{1}{4\pi} \cdot \frac{2z^2-x^2-y^2}{(x^2+y^2+z^2)^{\frac{5}{2}}}$$

In the implementation developed of the above algorithm, multiple source and target blocks can be defined, and the effects of multiple source blocks on the ΔB_0 field in a given target block are summed in linear superposition. Each source block is also zero-padded in practice to double its size along each dimension to avoid Fourier ghosting artifacts.

The algorithm was validated via a simple simulation experiment in which a sphere of water (χ = -9.0 ppm) with a radius of 12 voxels was placed in a background region of air (χ = 0.4 ppm). Eight susceptibility source blocks were chosen for testing purposes, together encompassing the sphere, and two target VOI blocks were chosen, one including part of the sphere and the surrounding region and the other near the edge of the simulation region. The resultant ΔB_0 field in these blocks was calculated and compared to the ΔB_0 field calculated using the existing calculation method. All simulation experiments were performed in MATLAB (The MathWorks, Inc., Natick, MA).

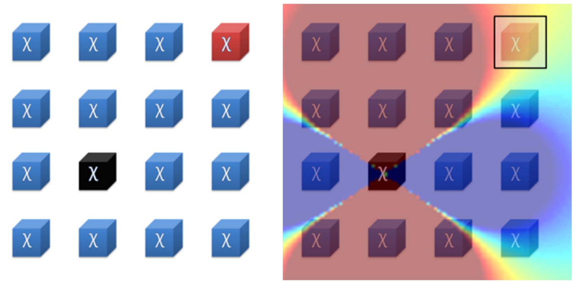

Figure 1. Diagram illustrating the concept behind the VOI-based Fourier Transform method. To calculate the effects of a source susceptibility block (black) on the ΔB_0 field in a target block (red), the portion of the image domain Green's function originating from the source block and overlapping the target block (black outline) is convolved with the source susceptibility block (black).

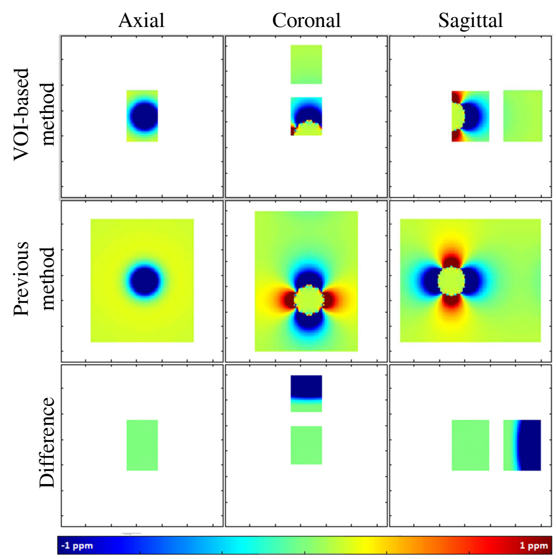

Figure 2. The new VOI-based ΔB_0 calculation method produces results identical to the previous method without Fourier ghosting artifacts. The difference map scaling is 100 times finer.

Results: Figure 2 shows the results of the simulation experiment. The new VOI-based ΔB_0 calculation method produces results identical to the previous method towards the center of the simulation region. At the edge, there is a large difference because the previous method suffers from Fourier ghosting artifacts at the edges.

Discussion: The new VOI-based ΔB_0 calculation method produces the same results as the previous calculation method without Fourier ghosting artifacts. The previous method can avoid such artifacts via zero padding to ensure that the FOV is large enough for the chosen susceptibility sources and VOIs, but this may result in even greater computation times and computer memory (RAM) usage. The drawback of the new method is that only a relatively limited volume of source and target blocks (~25% of the total volume) may be chosen before the computation time begins to exceed that of the previous method. The new method is therefore well-suited to "sparse" matrices wherein the susceptibility sources and VOIs may be relatively small and separated by relatively large distances. We used this method to iteratively calculate the ΔB_0 effects of sets of passive shim elements [4] and found a 50% reduction in computation time, which has a large impact for repeated, iterative calculations or "on-line" calculations that must be performed while the subject is still inside the magnet. The method may also be useful for high-resolution calculations of the ΔB_0 effects of one region of the body on another as calculating the whole body matrix at once as per the original method could require upwards of 10 GB of RAM and several minutes of computation time for very large matrices (e.g. 512 x 512 x 512 or greater) on a modern workstation.

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References: [1] Salomir et al. Concepts MR Part B (MR Eng'g) 19B (2003) 26–34 [2] Cheng et al., Phys Med Biol 54 (2009) 1169–89 [3] Lee and Hancu. JMRI 36 (2012) 873–80 [4] Dewal et al., Proc. ISMRM (2013), p. 5142