Using Spatio-Temporal Duality for Memory-efficient Non-uniform Fourier Transformation with Field Correction

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Target audience

Scientists concerned with non-Cartesian parallel image reconstruction

Introduction

Reconstruction of non-Cartesian parallel imaging data can be very challenging, particularly when big imaging matrices, off-resonance correction and/or non-linear reconstruction strategies coincide. Random undersampling in the non-Cartesian context in particular, gives rise to increased numeric complexity such that reconstruction times for typical whole head acquisitions easily exceed hours. However, Bloch dynamics as well as Faraday's law possess spatiotemporal symmetries, which can be exploited by time-reversing the physical process of signal acquisition to reconstruct images, implicitly including correction for off-resonances, within limits [1]. Although such a reverse solution of the Bloch equation is *per se* slower than non-uniform FFT solutions, the race is usually tied when typical off-resonances are considered. Additionally, such an inverse dynamics calculation is inherently massively parallel and thereby consuming diminishing amount of memory. These features suggest its implementation on graphics hardware (GPU) and distributed memory systems. Through parallelisation, speedups of orders of magnitude are realisable when compared to modern multithreaded libraries. C++ MPI and OpenCL codes of this reconstruction algorithm are made available along with test data on codeare.org.

$$\begin{aligned} s_n(t) &= \mathrm{i}\gamma m_0 \sum_{m=1}^{N} \int_0^T b_m(\tau) \\ &\quad \times \left(\int_{\mathbb{R}^3} \sigma_n^*(\mathbf{r}) \sigma_m(\mathbf{r})\, \mathrm{e}^{\mathrm{i}(\mathbf{k}(T-t)-\mathbf{k}(\tau))\mathbf{r}}\, d\mathbf{r} \right) d\tau \\ &= \mathrm{i}\gamma m_0 \sum_{m=1}^{N} \int_{\mathbb{R}^3} \frac{b_m(\mathbf{k})}{J(\mathbf{k})}\, \hat{\sigma}_{nm}\left(\mathbf{k}(T-t)-\mathbf{k}\right) d\mathbf{k} \end{aligned} \tag{1}$$

Equation 1: Hypothetical received signals, s_n, by coils n from a parallel-transmit excitation with coils m (by design complex conjugates). Further, b_m denote excitation pulses through channels m, k depicts the same trajectory for receive as well as transmit except of time-reversal.

Theory

Recently, we proposed the exploitation of MRI-inherent spatiotemporal symmetries for computation of small tip-angle pulses [1] in parallel transmission. The Bloch equation, neglecting the dissipative effects of T_1 and T_2*, which is an autonomous ordinary differential equation, and the reciprocity principle [4], the consequence of Faraday's law for MRI, display spatiotemporal symmetries that allows one to consider signal acquisition and small tip angle excitation [5] along the very same trajectory as a Fourier pair in parallel MRI (Eq.1). The operation that transforms a shape $\mathbf{m}(\mathbf{r})$ into signals $s_n(t)$, is the signal acquisition or the Fourier analysis and the inverse operation is the Fourier synthesis or small tip angle excitation. While the strict validity of reciprocity only applies to the stationary frame, i.e. transmit and receive sensitivities product $\sigma^*_\mathbf{n}\, \sigma_\mathbf{m} \neq \delta(n-m)$, one is free to design simulations where the rules are bent accordingly.

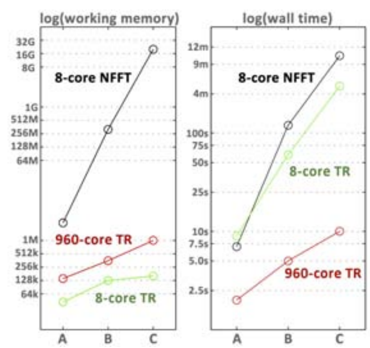

Figure 1: Comparison of runtimes and working memory consumption for 3 different datasets and according grid sizes. A: 2D 128^2, B: 3D 128^3, C: 3D 512^3. The reconstructions were performed with NFFT on 8 CPU cores (black line), with the proposed method on 8 CPU cores (green line) and on 960 GPU cores (red line).

Methods

To use this theory for image reconstruction is the next logical step. A measured MRI signal dataset is used in a hypothetical excitation simulation designed to restore the reciprocity yielding the excited pattern as the reconstructed image.

The algorithm is particularly well suited for parallel imaging as, within constraints of the small tip-angle approximation, the local RF field corresponds to a weighted sum of pulses from individual transmitters. This linearity constraint can again be easily imposed on a simulation. According to [6] for non-Cartesian Nyquist acceleration the Hermitian operator is needed as well, which conveniently is the simulation of the signal acquisition part of the very experiment used to obtain the data at hand.

Computational parallelisation can be achieved for forward and inverse operators with Nr x Nt and Nr, correspondingly; where, Nc, Nr and Nt denote the number of channels, image matrix sites and k-space sample points, respectively. Additionally, it should be noted that consideration of b_0 and b_1 do not increase the complexity of the algorithm, as the kronecker product of the sensitivity matrix with the encoding matrix is never actually performed.

Note further, that the simulator only covers the acquisition and excitation modules and the flexibility and completeness of simulators like jemris [7] is not required. A simple MRI "simulator" using rotation matrices computed in the Cayley-Klein manner [8] was programmed for CPUs with MPI and OpenMP parallelisation for C++ and MATLAB. Additionally, an MPI/OpenCL version was programmed for production on GPGPUs for real-time applications.

Results

Figure 1 shows the runtimes and memory consumption for different parallelisation schemes and data sets to illustrate the favourable scaling with respect to these variables. Figure 2 shows a typical "money-shot" reconstructed using the XXX method.

Figure 2: Coronal slice of a 2D 24-shot spiral scan 32-channel acquisition with b_0 correction (+/- 40Hz) from 3T.

Discussion and Conclusion

The MRI machine and sequence traditionally Fourier transform a weighted magnetisation density function as an analogue computer while the appropriate image reconstruction applies the inverse transform. In contrast to reality, where the reciprocity principle is violated at any Larmor frequency, a perfect world can be designed in simulation for reversing MRI signal formation process for image reconstruction. The proposed method is conceptually simple and opens up a range of potentially very useful instruments. One of the many applications would be to reconstruct parts of the image by restricting the spin population used for simulation to a region of interest. Also one can reconstruct images in a progressing fashion by simulating more and more dense grids until convergence is reached thus optimising the reconstruction to the resolution capabilities of the k-space sampling trajectory and to real-time needs. This study has demonstrated that an algorithmic inversion of the MRI signal formation process shows favourable performance for image reconstruction particularly in the case of non-Cartesian imaging in the presence of off-resonances, but more strikingly exhibits an orders of magnitude smaller memory footprint in parallel operation.

References

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