Optimal phase sensitive combination of multi-channel, multi-echo images

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Introduction

Magnetic resonance imaging has increasingly moved towards using multiple receiver channels. Because the coil sensitivity is often not available, the de facto standard for combining multiple receivers has become square root of the sum of squares, which inherently discards the signal phase making it unsuitable for phase sensitive processing. Phase is becoming increasing important for applications such as susceptibility weighted imaging, quantitative susceptibility mapping (QSM), and B_0 mapping. Many of these techniques are of particular interest at ultra high field where body coils that can be assumed to have uniform coil sensitivity are not available. This has spawned a growing body of literature on methods to combine multiple receivers in a phase sensitive manner[1-4]. Combination techniques proposed include simple global phase alignment[1], phase corrections relative to a reference coil or combination of coils[2], and referencing the phase against the first echo[3]. B_0 mapping and QSM frequently use multi-echo images, and these techniques utilize the available data in multi-echo acquisitions sub-optimally. For multi-echo data sets, it can be shown that the singular value decomposition SVD allows a best estimate, in a least squares sense, of the desired magnetization image by simultaneously estimating the magnetization and coil sensitivity.

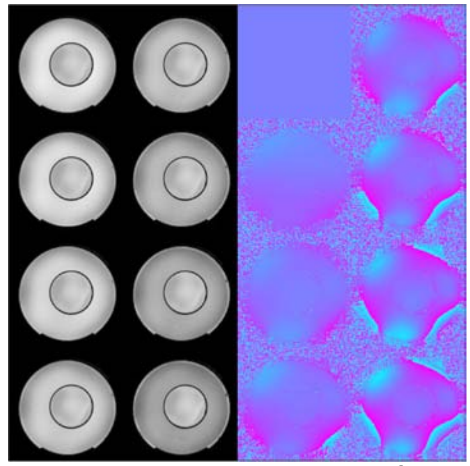

Figure 1 Magnetization magnitude (left) and phase (right) of the eight echoes.

Theory

For each location in space, the coil sensitivity **C** is a $N \times 1$ vector where N is the number of receiver coils. The magnetization **P** is a 1 x M vector where M is the number of echoes being reconstructed. Then the measured signal **S** is an $N \times M$ matrix where **S** = **CP**. This matrix is by definition a rank one matrix and the best estimate of **C** and **P** can be obtained using the SVD, assuming the channels have equal and uncorrelated noise. If the noise is unequal or correlated, then **S** should be transformed using an estimate of the $N \times N$ noise covariance **V** such that $\mathbf{S'} = \mathbf{V}^{-1}\mathbf{S}$, to achieve the requirement of equal and uncorrelated noise. Given the SVD $\mathbf{S'} = \mathbf{U}\,\boldsymbol{\Sigma}\,\mathbf{V}^{H}$ where $\boldsymbol{\Sigma}$ is the diagonal matrix of singular values, λ, ordered in decreasing value, i.e. λ_1 is the largest singular value. Then the first left eigenvector, i.e. first column of **U,** is an estimate of coil sensitivity $\tilde{\mathrm{C}}$ and the first right eigenvector, i.e. first column of **V,** is an estimate of the magnetization $\tilde{\mathrm{P}}$, and the best rank one estimate of the signal is $\lambda_1\tilde{\mathrm{C}}\tilde{\mathrm{P}}$. There is a constant arbitrary phase offset for $\tilde{\mathrm{C}}$ and $\tilde{\mathrm{P}}$ since $\tilde{\mathrm{C}}\tilde{\mathrm{P}} = \left(e^{i\theta}\tilde{\mathrm{C}}\right)\left(e^{-i\theta}\tilde{\mathrm{P}}\right)$. Therefore the first element of $\tilde{\mathrm{P}}$ can be assigned to be positive and real, i.e. zero phase. Because phase differences between images are generally of interest in multi-echo data, this is not typically an issue. The SVD is also defined such that the eigenvectors are unit length, i.e. $\tilde{\mathbf{C}}^H\tilde{\mathrm{C}} = 1$ and $\tilde{\mathbf{P}}\tilde{\mathbf{P}}^H = 1$, which means that the magnitude of **C** and **P** are contained in $\lambda_1 = (\mathbf{C}^H\mathbf{C}\mathbf{P}\mathbf{P}^H)^{1/2}$. Under the ubiquitous assumption that $\mathbf{C}^H\mathbf{C}$ is uniform, $\tilde{\mathrm{P}}$ should be scaled by λ_1 to obtain the desired magnetization images. This gives an estimate of **P** which contains the familiar $(\mathbf{C}^H\mathbf{C})^{1/2}$ scaling that is present in the standard square root of the sum-of-squares, and in the case of a single echo, it is equivalent.

Results

3D multi-echo FLASH (TR=19.1ms,FA=11°,TE=2.1ms,ESP=1.3ms) with 8 bipolar echoes and 1.4 mm isotropic resolution were acquired using a 16-channel cylindrical symmetric transceive RF coil where acquired on an 7T human system. The magnitude and phase images of the eight echoes, $\lambda_1\tilde{\mathrm{P}}$ (Figure 1), as well as the sixteen coil sensitivity maps, $\tilde{\mathrm{C}}$ (Figure 2), where generated by using a simple point-by-point SVD.

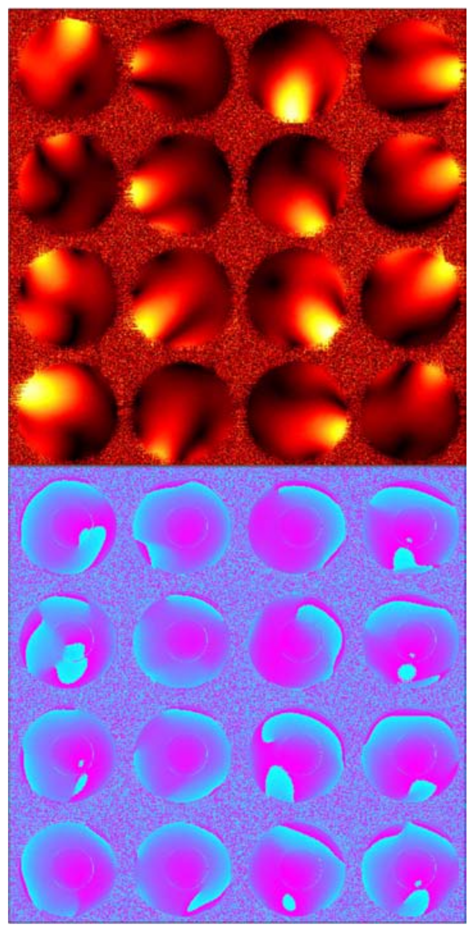

Figure 2 Coil sensitivity magnitude (top) and phase (bottom) of the sixteen coils.

Conclusion

Optimal, in a least squares sense, phase sensitive coil combination of multi-echo data can be obtained using a point-by-point SVD. This method is similar to the stochastic method proposed by Walsh et al[4]. Since coil sensitivities are acquisition independent, they could be computed on a low-resolution prescan and interpolated for use in reconstructing any other acquisitions.

References

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