A Bilinear Noise Transfer Model for SENSE Reconstruction

Yu Ding[1], Rizwan Ahmad[1], Hui Xue[2], Lee Potter[1], Samuel T Ting[1], Ning Jin[3], and Orlando P Simonetti[1]
[1]*The Ohio State University, Columbus, OH, United States,* [2]*Corporate Research, Siemens Corporation, Princeton, NJ, United States,* [3]*Siemens Healthcare, Columbus, OH, United States*

Introduction: SENSE is a widely used parallel imaging technique that has a strict closed-form solution [1]. It is known that the so-called g-factor represents how noise in the raw data affects the noise in the reconstructed image. However, the g-factor calculation does not take into account the noise in the channel sensitivity map. In other words, the traditional g-factor calculation assumes a perfect channel sensitivity map, which is not available in practice. In this abstract, we present a noise transfer model in SENSE reconstruction that takes into account the noise in both the raw data and the channel sensitivity map. We validate the proposed bilinear model using images acquired in a phantom, and evaluate the potential impact that noise in the channel sensitivity map may have on SENSE reconstructed images.

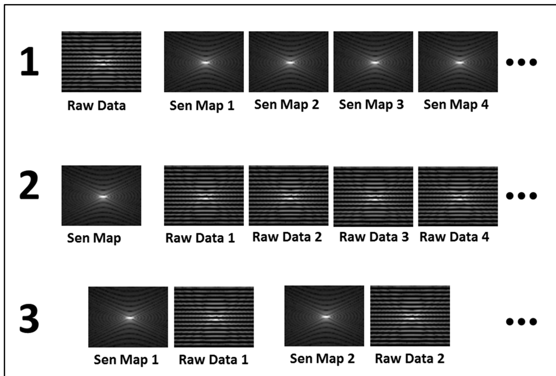

Figure 1. Diagram of three image series reconstructed from the same set of k-space raw data.

Theory: SENSE reconstruction is a linear process:

$$V = \boldsymbol{E}^{-1}M \qquad (1)$$

where V is the reconstructed image, M is the wrapped image (raw data), $\boldsymbol{E}$ is the SENSE encoding matrix, and $\boldsymbol{E}^{-1}$ is the (pseudo)-inversion of $\boldsymbol{E}$. Both $\boldsymbol{E}$ and M have noise, *i.e.* $\boldsymbol{E} = \boldsymbol{E}_0 + \boldsymbol{N}_E$, and $M = M_0 + N_M$, where $\boldsymbol{N}_E$ and N_M are the noise in the encoding matrix (from channel sensitivity map) and wrapped image, respectively. When entries of $\boldsymbol{N}_E$ are small compare to $\boldsymbol{E}_0$, then

$$\boldsymbol{E}^{-1} \approx \boldsymbol{E}_0^{-1} - \boldsymbol{E}_0^{-1}\boldsymbol{N}_E\boldsymbol{E}_0^{-1} \qquad (2)$$

Therefore, Eq. (1) becomes:

$$V \approx (\boldsymbol{E}_0^{-1} - \boldsymbol{E}_0^{-1}\boldsymbol{N}_E\boldsymbol{E}_0^{-1)}(M_0 + N_M) \approx \boldsymbol{E}_0^{-1}M_0 - \boldsymbol{E}_0^{-1}\boldsymbol{N}_E\boldsymbol{E}_0^{-1}M_0 + \boldsymbol{E}_0^{-1}N_M \qquad (3)$$

where $\boldsymbol{E}_0^{-1}M_0$ represents the ideal reconstructed image, $\boldsymbol{E}_0^{-1}\boldsymbol{N}_E\boldsymbol{E}_0^{-1}M_0$ and $\boldsymbol{E}_0^{-1}N_M$ represent the noise from sensitivity map and raw data, respectively. Therefore, there are three types of noise in the reconstructed image: the noise from sensitivity map, the noise from raw data, and the total noise, with standard deviations σ_s, σ_d, and σ_t. We assume that noise from sensitivity map and noise from raw data are completely uncorrelated, then

$$\sigma_t = \sqrt{(\sigma_s^2 + \sigma_d^2)} \qquad (4)$$

Equation 4 can be used as a self-consistent check of the bilinear noise transfer model.

Methods: We acquired one series of SSFP real-time cine image on a 1.5T MR scanner (MAGNETOM Avanto, Siemens Healthcare, Germany) using a standard Siemens 12-channel phased-array receiver coil. Imaging parameters were: 160 × 120 matrix, 8 mm thick slice, flip angle= 70^o, TE/TR = 1.0/2.4 ms, pixel bandwidth=1488 Hz/pixel, FOV = 400 × 300 mm^2. A total of 81 images per image series with fully sampled k-space were acquired to support statistical analysis. All images were reconstructed offline on a personal computer (Intel Duo Core Quad 3.0GHz CPU, 16GB memory) using software written in MATLAB.

σ_s, σ_d, and σ_t were measured in the temporal direction of the following three reconstructed image series (refer to Fig. (1)): 1) identical raw data, but a different sensitivity map used to reconstruct each frame (i.e. down-sample the first frame as the input raw data, use the remaining 80 frames to estimate 80 sensitivity maps, and reconstruct 80 frames using the raw data and each sensitivity map); 2) identical sensitivity map, but different raw data used to reconstruct each frame (i.e. use the first frame to estimate sensitivity map, down-sample the remaining 80 frames, and reconstruct the remaining 80 frames); 3) both a different sensitivity map and different raw data used to reconstruct each frame (i.e. group 80 frames into 40 pairs, use the first frame in each pair to estimate sensitivity map, down-sample the second frame in each pair as the input raw data, and reconstruct 40 frames). The noise levels of all three image series was evaluated using the MP-law method [2].

Results: The experimental results are summarized in Table I. The left-hand side and the right-hand side of Eq. (4) are listed in columns 4 and 5, and column 6 is the relative error between them. The small error in the results indicates good accuracy of the model. At any acceleration rate, the noise from the sensitivity map exceeds 35% of the noise from the raw data and, therefore, is not negligible.

Discussion and Conclusion: We proposed a novel bilinear noise transfer model to investigate how the noise in the sensitivity map and raw data affect the noise in SENSE reconstructed images. A phantom study showed that the model has satisfactory accuracy, and with the help of the model, we observed that a large portion (> 35%) of the image noise originates from the noise in the sensitivity map, especially at higher acceleration rates. Our study suggested that a high SNR sensitivity map estimate will improve the SENSE reconstruction significantly.

In addition, our study showed that temporal noise measurements [2, 3] may under-estimate the spatial noise level, e.g. if a common sensitivity map is used for a dynamic image series, then the spatial noise introduced by sensitivity map does not appear in the temporal direction. Therefore, special care must be taken when assessing noise level by measuring across images in the temporal direction.

References: [1] Pruessmann, KP et al, Magn Reson Med 42: (1999) 952. [2] Ding, Y et al, Magn Reson Med 63: (2010) 782. [3] Robson, PM et al, Magn Reson Med 60: (2008) 895.

Table I

Acc. Rate	σ_s	σ_d	σ_t	$\sqrt{(\sigma_s^2 + \sigma_d^2)}$	Error
2	0.2508	0.3896	0.4505	0.4634	2.86%
3	0.5295	0.6406	0.8147	0.8311	2.01%
4	1.2217	1.3754	1.8080	1.8396	1.75%
5	3.2909	3.2560	4.1531	4.6294	11.5%