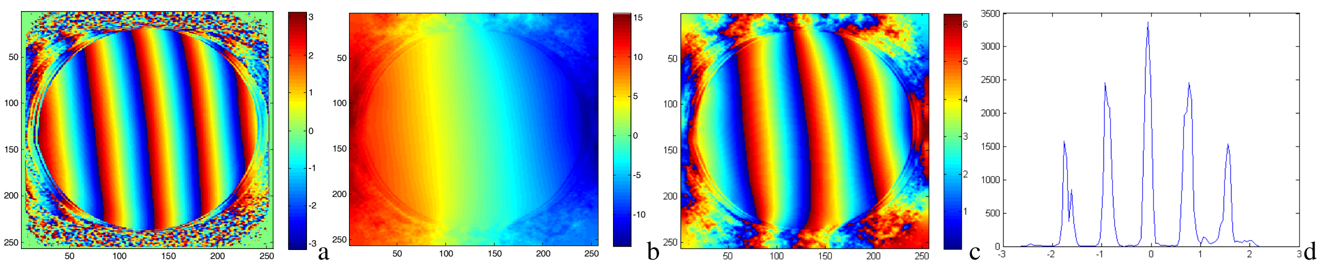
AN EXACT PHASE UNWRAPPING METHOD BASED ON FFT

Renjie He[1], Joshua P. Yung[1], David T. A. Fuentes[1], Jason R. Stafford[1], and John D. Hazle[1]
[1]Department of Imaging Physics, The University of Texas M. D. Anderson Cancer Center, Houston, Texas, United States

Purpose: 2D/3D phase unwrapping are of increasing importance in many application including MR thermal imaging as well as susceptibility weighted imaging [1]. Though there are many phase unwrapping approaches, 2D/3D phase unwrapping based on FFT is attractive due to its flexibility and speed [2][3]. However, in practice, the unwrapped phase obtained directly from FFT could be distorted so that the phase does not meet with the condition of $2\times N\times\pi$ + "the wrapped phase" [4] (see Figure 1 for example, where (a) is the wrapped phase, and (b) is the directly unwrapped phase from FFT), where N is an integer field. It is noticeable that for most of the phase unwrapping methods, the optimization is not for retrieving the integer field since integer optimization is too costly; therefore, many of the methods will not be able reconstruct an unwrapped phase exactly satisfying $2\times N\times\pi$ + "the wrapped phase". Usually the break of the integer condition will result in the histogram of the integer field N becoming dispersed as a real valued field (see Figure 1 (d) and (e)); therefore, to correct the distortion of the unwrapped phase, we need to recover the integer field N from the real valued field.

Method: Based on Figure 1 (d), we realize that an optimization procedure is needed to quantize the real valued field (see Figure 1 (e)) into the integer field consisting of piece-wise consecutive integers (see Figure 1 (f)). To complete the integer quantization, Powell optimization is applied [5] where the cost function is simply formulated as the absolute difference between the wrapped phase (e.g. Figure 1 (a)) and the re-wrapped phase (e.g. Figure 1 (c)). The variables are the consecutive real-valued intervals to be mapped into integer intervals, and the integer quantization is completed on the real-valued field (e.g. Figure 1 (e)). Since no gradient is available for such cost function, Powell optimization is adopted so that the minimization is merely based on evaluations of the cost function. Once the integer field is recovered (e.g. Figure 1 (g)), we can reconstruct the exactly unwrapped phase by applying $2\times N\times\pi$ + "the wrapped phase". This procedure may be helpful for other phase unwrapping methods whenever the exact phase unwrapping is required for quantitative analysis.



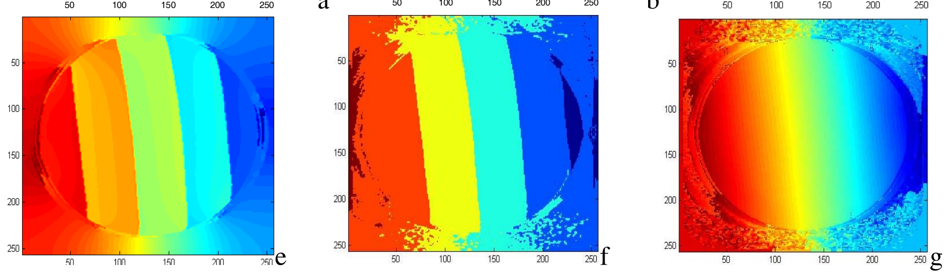


Fig. 3 The phase unwrapping based on FFT. (a) the wrapped phase image, (b) the directly unwrapped phase, (c) the re-wrapped phase of (b), (d) the histogram of the dispersed integer field, (e) the dispersed integer field acquired by FFT method, (f) the integer filed after applying Powell optimization on (e), (g) the exact unwrapped phase based on the results of (f).

Results: Phantom phase MR image is acquired as shown in Figure 1 (a). Using FFT based phase unwrapping methods [2][3], the distorted unwrapped phase as well as the dispersed integer field are acquired as shown in Figure 1 (b) and (e). The Powell optimization is used to quantize the dispersed integer field into the integer field as shown in Figure 1 (f), and, using the integer field, the exactly unwrapped phase image can be obtained as shown in Figure 1 (g).

Conclusions: FFT Based phase unwrapping is a straight, flexible and fast method; however, it suffers from the phase distortion in practice. We tried to recover the phase unwrapping exactly by quantizing the dispersed integer filed into an integer field through Powell optimization. The exact phase unwrapping was obtained for phantom phase MR images. This approach is useful in quantitative analysis based on phase images.

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