Using Extended Phase Graphs: Review and Examples

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Purpose: The extended-phase-graph (EPG) formalism [1-3], which allows efficient signal simulation for numerous commonly used pulse sequences, is summarized intuitively, with applications and software.

Basics: EPG decomposes magnetization within a voxel into a basis of transverse (F_n) and longitudinal (Z_n) states, as shown in Fig. 1, where n is the number of "twists" or cycles across a voxel. Assuming all gradients dephase an integer number of cycles across a voxel and that spins are homogeneously distributed in the voxel, this basis is perfectly invertible. Because M_{xy} is complex and M_z is real, F_n is defined for positive and negative n, while Z_n is defined for $n \geq 0$.

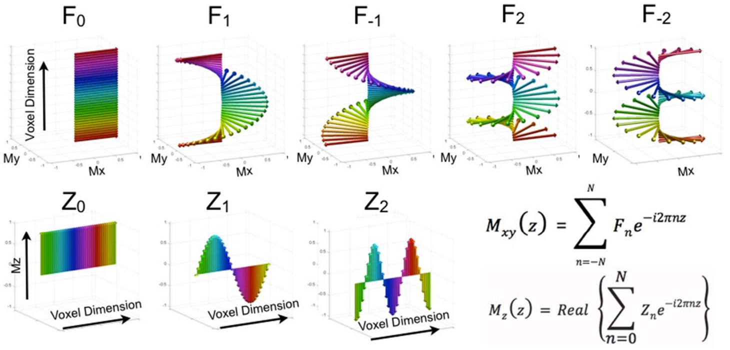

Figure 1: *EPG basis for M_{xy} and M_z: F_n states consist of twists of n cycles across a voxel, in opposite directions for positive and negative n. Z_n states consist of sinusoidal variation of M_z ($n \geq 0$). Equations show simple Fourier transform relationship between EPG states and magnetization.*

Sequence Propagation: During each TR, precession due to gradients or off-resonance causes the transverse magnetization to evolve to the next dephased state (Fig. 2), while magnetization in longitudinal states remains there until it is transferred to a transverse state by an RF pulse. Relaxation causes decay of transverse and longitudinal states, and recovery of the Z_0 state. RF pulses transfer magnetization between states of order n (F_n, F_{-n} and Z_n). Diffusion attenuates states in proportion to $D \times n^2$, as well as the square of the *actual* gradient area. Using simple building blocks, complicated sequences can be built up easily, and at any time, the F_0 state corresponds to the observed signal.

Examples: Any sequence with integer dephasing across a voxel over a repetition can be simulated easily with EPG. Example of gradient-echo sequences are gradient-spoiled (FISP, GRE) sequences [4], RF-and-gradient spoiled (FLASH, SPGR, T1-FFE), reverse gradient-spoiled (PSIF, CE-FAST), and stimulated echo (STEAM). Spin-echo sequences with CPMG, non-CPMG, reduced and varying refocusing flip angles can all very easily be built up [5]. Diffusion effects can be simulated in all cases very easily, using the formulation provided by Wiegel et al. [6]. Matlab code and examples are at http://bmr.stanford.edu/epg.

Discussion Points:

The EPG formalism allows efficient simulation of groups of spins within a voxel by transforming magnetization into a Fourier basis, which is very compact when gradients dephase an integer number of cycles over a voxel.

- Magnetization can easily be Fourier transformed between the EPG $\{F_n, Z_n\}$ basis and a spatial spin distribution $\mathbf{M}(\mathbf{r})$, allowing modeling in two domains.
- Static dephasing effects can be modeled with EPG, then transforming to $\mathbf{M}(\mathbf{r})$. Each position in $\mathbf{M}(\mathbf{r})$ corresponds to a specific amount of phase accrual per unit time.
- Higher states can be eliminated from simulation when their content approaches zero.
- Assuming N non-zero states, the EPG propagations from relaxation, RF nutation, gradients and diffusion can be expressed as a linear system, so that complicated steady states can be quickly calculated numerically.
- Motion can be modeled with constant (bulk) phase shifts at gradient steps.
- EPG can be extended to model higher-dimensional gradient dephasing.

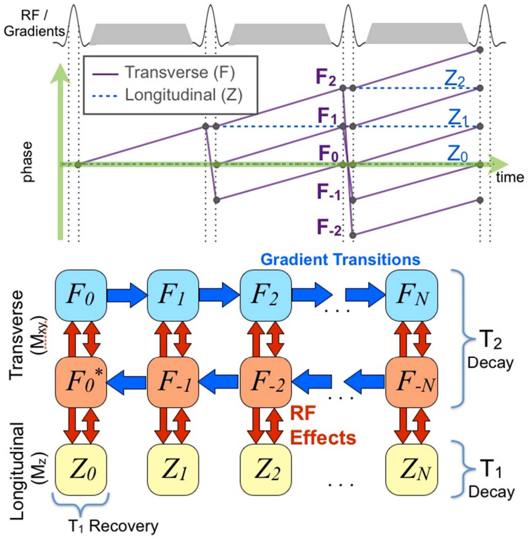

Figure 2: *Alternate illustrations of magnetization propagation. (top) Coherence pathways showing how magnetization reaches different F and Z states during a sequence of RF and gradient pulses. (bottom) 3xN Matrix representation of F and Z states with transitions due to RF pulses (vertical mixing of F and Z states), gradients (shifts in F states) and relaxation (decay of all states, plus recovery in Z_0 only). Gradients of opposite sign would shift F states in the other direction.*

Summary: The EPG formulation offers a powerful and simple means to simulate numerous pulse sequences and effects in MRI. The reader is encouraged to explore the tools and examples to gain understanding of this powerful technique.

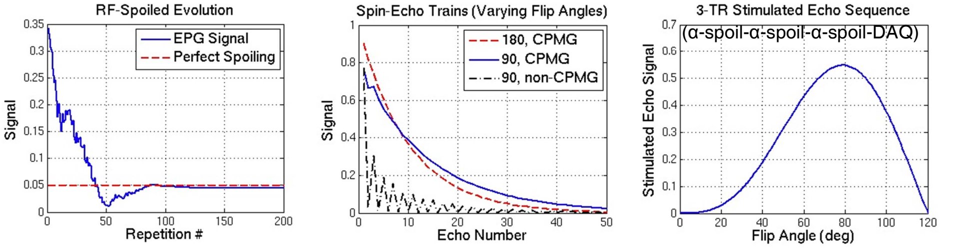

Figure 3: *Example simulations using EPG Matlab code. Each requires a few simple lines of code, and easily completes in a fraction of a second, where Bloch simulations may be more challenging.*

References: [1] Hennig J. JMR 78(3) p.397, 1988. [2] Hennig J. Concepts in MR 3(3) p.125, 1991. [3] Hennig J. Concepts in MR 3(4) p.179, 1991. [4] Scheffler Concepts in MR 11 p.291, 1999. [5] Hennig J, et al. 51(1) p.68, 2004. [6] Weigel M et al. JMR 203, p.276, 2005.