Analytic solution of the optimum flip angle for pass-band SSFP fMRI prescribes high flip angle acquisitions

Steve Patterson[1,2] and Chris Bowen[1,2]

[1]*Dalhousie University, Halifax, NS, Canada,* [2]*Institute for Biodiagnostics (Atlantic), NRC, Halifax, NS, Canada*

Introduction Pass-band balanced steady state free precession (pass-band SSFP) is growing in popularity as a functional MRI (fMRI) technique because it offers reduced signal dropout and image distortion compared to gradient-recalled-echo echo-planar-imaging (GRE-EPI), while potentially providing greater blood oxygenation level dependent (BOLD) sensitivity than spin-echo acquisitions. Most pass-band SSFP fMRI studies employ the signal-optimizing flip angle (α_s), given by[1]:

$$\cos(\alpha_s) = ((T1/T2)-1)/((T1/T2)+1) \qquad (1)$$

In this work we derive an analytical expression for the BOLD contrast-optimizing flip angle (α_c), under the assumption that on-resonant pass-band SSFP BOLD contrast results only from R_2 changes, with no contribution from frequency shifts[2]. Interestingly, in grey matter at 3T α_c is about 20° larger than α_s. Validating our result against Monte Carlo simulations (which do include frequency shifts), we found that the use of α_c rather than α_s provided 23% more BOLD contrast on-resonance, as well as more uniform BOLD contrast off-resonance.

Theory The on-resonant SSFP signal, S, at $T_E=T_R/2$, is given by[2]:

$$S = A\sin(\alpha)/(B\cos(\alpha) + C) \qquad (2)$$

where α is the flip angle, $A = \mathrm{sqrt}(E_2)(1-E_1)$, $B = -(E_1-E_2)$, $C = 1-E_1E_2$, $E_1 = \exp(-R_1T_R)$, $E_2 = \exp(-R_2T_R)$, $R_1 = 1/T_1$, and $R_2 = 1/T_2$. Approximating BOLD activation as a change in the R_2 relaxation rate, we express SSFP BOLD contrast as:

$$\Delta S = S(A_a, B_a, C_a) - S(A_r, B_r, C_r) \qquad (3)$$

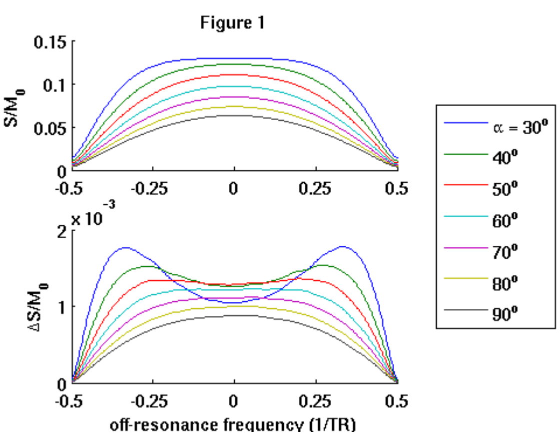
where $A_r = A(R_1, R_{2,rest})$, $A_a = A(R_1, R_{2,active} = R_{2,rest}+\Delta R_2)$, etc. Taking the derivative of this expression with respect to flip angle and setting it to zero results in the cubic equation:

$$ax^3 + bx^2 + cx + d = 0 \qquad (4)$$

where $x = \cos(\alpha)$, $a = (A_aC_aB_r^2-A_rC_rB_a^2)$, $b = (2A_aC_aC_rB_r+A_aB_aB_r^2-2A_rC_rC_aB_a-A_rB_rB_a^2)$, $c = (2A_aB_aC_rB_r+A_aC_aC_r^2-2A_rB_rC_aB_a-A_rC_rC_a^2)$, and $d = (A_aB_aC_r^2-A_rB_rC_a^2)$. Solving for the roots of this equation, and choosing the root satisfying $|x| <= 1$ that is required for a real flip angle, gives an analytic expression for α_c of the form $\alpha_c = f(T_1, T_2, T_R, \Delta R_2)$.

Methods We compared the analytical solution with Monte Carlo simulations of SSFP fMRI contrast at 3T, following established approaches[2,3] shown to agree with experiment. Blood vessels were modeled as randomly oriented cylinders having a blood-oxygenation-dependent magnetic susceptibility offset from their surroundings. A simplified grey matter model was used[3] consisting of 2% (by volume) radius (R) = 3μm vessels and 3% R = 100μm vessels. BOLD activation was simulated by changing the blood oxygenation saturation fraction from 0.67 (resting) to 0.75 (active)[2]. Vessels were embedded in a homogeneous medium having grey matter relaxation times (T_1 = 1200 ms, T_2 = 90 ms). Vessels were treated as impermeable. The intravascular compartment was included in the Monte Carlo model. Intravascular T_1 was set to extravascular, while intravascular T_2 was computed from a Luz-Meiboom exchange model fit to SSFP data from in-vitro blood samples at 3T[4].

Results In Figure 1 we plot the pass-band SSFP (T_R = 10 ms, $T_E = T_R/2$) normalized resting signal (S/M_0, top) and corresponding BOLD contrast ($\Delta S/M_0$) vs. off-resonance frequency from the Monte Carlo simulation at several flip angles. From Eq. 1, α_s = 31°, and from Eq. 4 (using a literature reported BOLD-induced ΔR_2 of $-0.4s^{-1}$ at 3T[5]), α_c = 51°. Using α_c rather than α_s, while reducing signal, results in both greater contrast in the pass-band centre (23% more in this example) and more uniform contrast across off-resonance frequency (6% variation over the pass-band (± $0.25/T_R$) region for α_c, vs. 43% for α_s). The contrast profile from α_c resembles the flat pass-band signal profile from α_s.



Discussion and Conclusion An order-of-magnitude estimate for ΔR_2 is sufficient to compute α_c. In the above example, α_c = 51° and 52° for $\Delta R_2 = -.01s^{-1}$ and $-1s^{-1}$, respectively, spanning a broad physiological range. The contrast-optimizing flip angle is largely determined by the T_1 and T_2 relaxation times. Using literature-reported relaxation times for grey matter[6], we found the relationship $\alpha_c \approx \alpha_s + 20°$ to give the contrast-optimizing flip angle to within 10% for B_0 from 1.5-7T. In conclusion, we have derived an analytic expression and formulated a simple rule-of-thumb for the pass-band SSFP fMRI contrast-optimizing flip angle, and have shown it to increase on-resonant BOLD sensitivity and provide more uniform BOLD sensitivity off-resonance.

References

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