Effects of noise estimation error on the accuracy and precision of maximum likelihood estimation of apparent diffusion coefficient

Jing Yuan[1], Qinwei Zhang[1], David Ka Wai Yeung[1], Anil T Ahuja[1], and Ann D King[1]

[1]*Department of Imaging and Interventional Radiology, Chinese University of Hong Kong, Shatin, NT, Hong Kong*

Purpose: The Rician noise distribution of magnitude MR image deviates considerably from Gaussian distribution at low signal-to-noise ratios (SNRs) [1]. This results in the biased estimation of imaging parameters when least-squares estimation (LSE) is used. Maximum likelihood estimation (MLE) [2] has been proposed for better apparent diffusion coefficient (ADC) mapping [3] at low SNRs associated with large ADCs and b-values. Noise standard deviation (σ) or variance (σ^2) estimation is essential for MLE but accurate σ estimation is hampered by many factors such as ghosting artifacts, multi-coil combination, inappropriate ROI selection and image filters (e.g. smoothing, denoising) applied to magnitude images. This may lead to the reduced estimation accuracy and precision of MLE. This study aims to investigate the effects of error in noise estimation on the estimation of ADC by using MLE so as to evaluate the robustness of MLE for ADC mapping in clinical practice.

Methods: Noiseless DWI signal was generated by $S=S_0\exp(-b*ADC)$ (with b-values of [0, 200, 400, 600, 800, 1000]s/mm^2 and ADC=0.001mm^2/s) and Rician noise was then added. Monte-Carlo simulations were performed to test the robustness of MLE when noise level σ is misestimated at different true SNRs (n=20,000 per simulation). SNR was defined as the DWI signal at b=0 divided by σ, i.e. S_0/σ. Statistics of estimation distributions of MLE were compared to LSE. MLE and LSE were also applied and compared for ADC mapping of clinical DWI images of head and neck tumors obtained with the same b-values as simulation on a 3T Philips Achieva scanner (single-shot fat-suppressed SE-EPI with TE/TR=42ms/2s, FOV/thickness=230/4mm, matrix=136x109). True σ was estimated from the Rayleigh noise distribution of carefully selected background ROIs without motion and ghosting artifact.

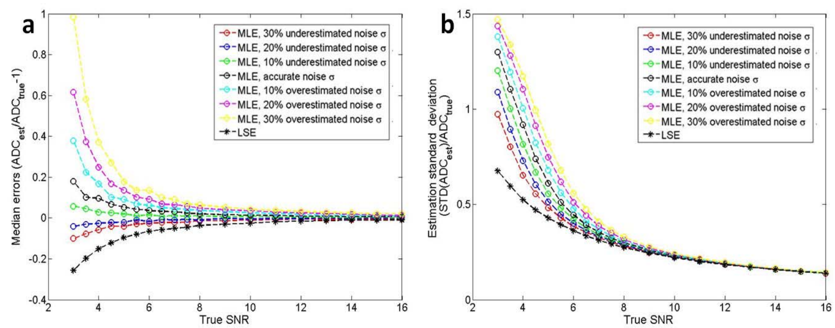

Fig.1. Monte Carlo Simulation results of median and standard deviation distribution of ADC using MLE and LSE.

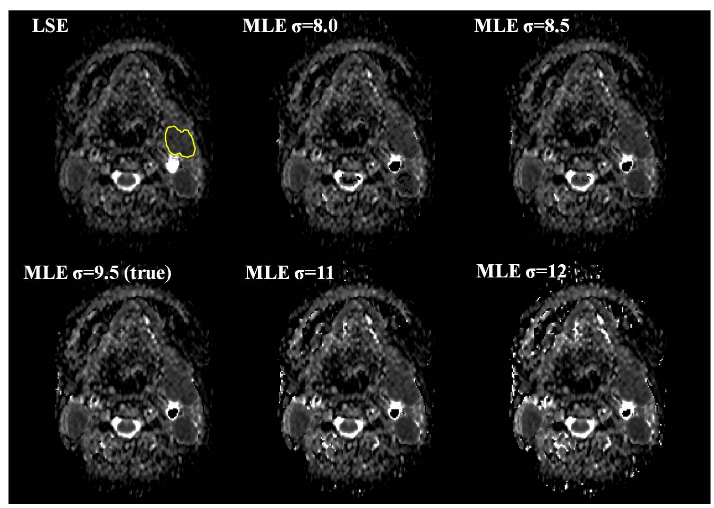

Fig.2. ADC maps of metastatic nodes (one is delineated by yellow ROI) of head and neck tumor by MLE and LSE. True noise σ≈9.5.

Results: Monte-Carlo simulation results (Fig.1) showed that LSE systematically underestimated ADC. MLE slightly underestimated ADC when σ underestimation was within 20%. Otherwise, ADC was overestimated with the increasing estimated σ. When true SNR>6, MLE achieved ADC deviation smaller than LSE. If true SNR>10, MLE exhibited relatively good robustness to erroneous noise estimate. On the other hand, the standard deviation of the estimated ADC by MLE increased with the estimated σ but was always larger than LSE (Fig.1b). ADC mapping comparison (Fig. 2) showed that MLE was fairly robust to noise estimation errors, in which case the true noise σ was about 9.5 and the SNR of one metastatic node (delineated by the yellow ROI on Fig. 2) was around 13. The statistics of the node ADC were listed in Table. 1. Consistent with simulation results, median and standard deviation of ADC increased with the estimated σ, and were always larger than corresponding values by LSE.

(*μm²/ms*)	LSE	MLE (σ=8)	MLE (σ=8.5)	MLE (σ=9.5)	MLE (σ=11)	MLE (σ=12)
Mean ADC	0.6373	0.6777	0.6841	0.6996	0.7326	0.7655
Median ADC	0.6146	0.6582	0.6648	0.6785	0.7072	0.7347
Standard Deviation	0.2875	0.3071	0.3103	0.3185	0.3378	0.3606

Table. 1. Statistics of one metastatic node ADC by using MLE and LSE

Discussion: Although MLE theoretically offers unbiased estimation of ADC from magnitude MR images, its accuracy and precision are still subject to noise level estimation error. Particularly at low SNRs, noise estimation error may lead to the significant reduction of MLE accuracy and precision, even worse than LSE. At relatively high SNRs, MLE shows fairly good robustness to noise estimation error and provides more accurate ADC mapping than LSE. This work is supported by HK RGC grant CUHK418811 and SEG_CUHK02. **References:** [1] Gudbjartsson H and Patz S, MRM 1995, 34:910-914; [2] Sijbers J et al, IEEE-TMI, 1998, 17:357-361; [3] Walker-Samuel S, et al, MRM 2009, 62:420-429;