Convenient b-value and b-matrix computations for arbitrary gradient waveforms: characterizing signal decay due to Gaussian diffusion

Evren Ozarslan[1] and Thomas H Mareci[2]

[1]*Department of Radiology, Brigham and Women's Hospital, Harvard Medical School, Boston, MA, United States,* [2]*Department of Biochemistry and Molecular Biology, University of Florida, Gainesville, FL, United States*

INTRODUCTION: Diffusion-induced attenuation of the MR signal leads to unique image contrasts, which have been of great value in both clinical and basic science applications of MRI. Such contrast emerges from variations in the diffusion decay rate, referred to as apparent diffusion coefficient (ADC) when diffusion is measured along one orientation, and apparent diffusion tensor (ADT) for the case of anisotropic diffusion. The amount of attenuation is controlled by the b-value and b-matrix, respectively. Accurate estimations of ADC and ADT are possible only when all parameters of the pulse sequence (gradient waveform) are taken into account in the b-factor estimations. In this work, we demonstrate that these b-factors can be computed very conveniently and efficiently for all pulse sequences using a few lines of a computer program without the need to employ analytical derivations specific to a given pulse sequence [1,2] or a symbolic mathematics software package [3] such as Maple or Mathematica.

THEORY: The b-value is given by the expression $b = \gamma^2 \int_0^T \left| \int_0^t \mathbf{G}(t')\, \mathrm{d}t' \right|^2 \mathrm{d}t$, where γ is the gyromagnetic ratio and T is the duration of the effective gradient waveform, $\mathbf{G}(t')$, which is obtained by incorporating the effects of radiofrequency pulses employed in the sequence. The key idea in our approach involves representing this waveform with a piecewise constant function with N time intervals so that $\mathbf{G}(t) = \sum_{n=1}^{N} \mathbf{G}_n(t)$, where $\mathbf{G}_n(t) = \begin{cases} \mathbf{g}_n, & T_{n-1} \le t < T_n \\ 0, & \text{otherwise} \end{cases}$. Figure 1 illustrates such a piecewise constant waveform. Note that $T_n = T$ and $\delta_n = T_n - T_{n-1}$. Inserting the expression for $\mathbf{G}_n(t)$ into the integrals, and carrying out the algebra, one obtains $b = \sum_{n=1}^{N} b_{nn} + 2\sum_{m=1}^{N-1}\sum_{n=m+1}^{N} b_{mn}$, where $b_{nn} = \gamma^2 \delta_n^2 |\mathbf{g}_n|^2 \left(T - T_{n-1} - \frac{2}{3}\delta_n\right)$ and $b_{mn} = \gamma^2 \delta_m \delta_n \mathbf{g}_m \cdot \mathbf{g}_n \left(T - T_{n-1} - \frac{1}{2}\delta_n\right)$ for $n > m$.

Essentially the same scheme can be employed for the estimation of each component of the b-matrix, resulting in very similar expressions. These expressions are not included here for brevity.

RESULTS: To establish the correctness of the above scheme and assess its accuracy, we computed the b-value for a cosine-modulated oscillating waveform. In Figure 2, we plot the percent deviation in our b-value estimates from its theoretical value [4] against the number of distinct time intervals when all intervals were taken to have the same duration. As expected, the error in the estimates decay rapidly to negligible levels as the number of intervals is increased. It should be noted that the instrument's gradient waveform generator produces a piecewise constant current in practice. Therefore, the b-value computed using our approach, with the appropriate choice of number of intervals, may be more accurate than its theoretical value, which assumes perfectly continuous waveforms.

DISCUSSION: One important application of this method for evaluating b-factors is in studies that aim to recover tissue's microstructural parameters from diffusion-weighted signals. Such studies tend to model diffusion within the intracellular space as restricted and diffusion in the extracellular matrix as a Gaussian process [5]. Although the problem of estimating signal attenuation due to restricted diffusion has been solved for piecewise constant gradient waveforms [6,7], no such solution had been reported for the significantly less challenging problem of Gaussian diffusion. Our scheme models Gaussian diffusion using essentially the same input required by [6,7]. As such, we envision this approach to be complementary to the widely employed solutions for restricted diffusion, thus yielding a complete and consistent modeling framework.

CONCLUSION: We presented a quick and convenient method for computing the b-value and b-matrix for an arbitrary gradient waveform. This solution could be employed to: (i) conveniently incorporate the effects of *imaging gradients* in traditional Stejskal-Tanner sequences, (ii) compute the signal attenuation for *more sophisticated diffusion encoding schemes* (e.g., schemes with oscillating waveforms, and multiple pairs of gradients), and (iii) *complement the existing solutions for restricted diffusion* in biophysical modeling studies.

REFERENCES: [1] Mattiello et al., Magn Reson Med, 37, p. 292, 1997. [2] Sinnaeve, Concepts Magn Reson Part A, 40, p. 39, 2012. [3] Güllmar et al., Concepts Magn Reson Part A, p. 53, 2005. [4] Gore et al., NMR Biomed, 23, p. 745, 2010. [5] Assaf and Basser, Magn Reson Med, 52, p. 965, 2004. [6] Grebenkov, Rev Mod Phys, 79, p. 1077, 2007. [7] Özarslan et al., J Chem Phys, 130, 104702, 2009.

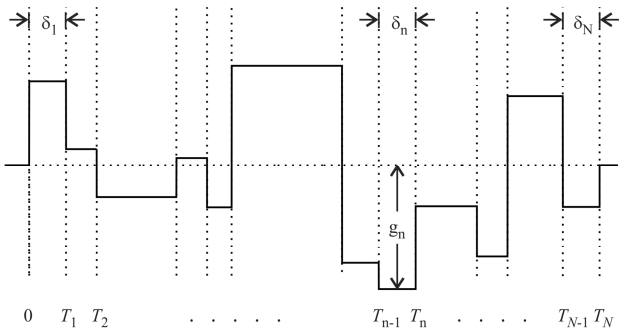

Figure 1: A piecewise-constant gradient waveform. When the gradient waveform is piecewise-constant, the b-value estimates are exact. Our method can be applied to smoothly varying waveforms by approximating them by piecewise constant profiles.

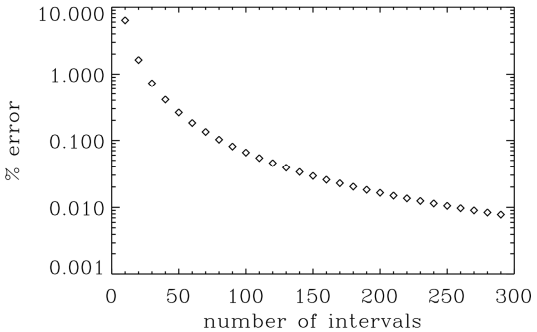

Figure 2: Percent error in the b-value estimates associated with approximating a cosine-shaped oscillating gradient waveform with a train of piecewise constant functions.