Random Matrix Theory-based noise reduction for dynamic imaging: Application to DCE-MRI

Jeiran Jahani[1], Glyn Johnson[2], Valerij G Kiselev[3], and Dmitry S Novikov[4]

[1]*New York University School of Medicine, New York City, New York, United States*, [2]*University of East Anglia, East Anglia, United Kingdom*, [3]*University Medical Center Freiburg, Freiburg, Germany*, [4]*Radiology, New York Univeristy School of Medicine, New York City, New York, United States*

1.Purpose: How to denoise dynamic MRI maps? If the maps are static, e.g. a diffusion or relaxation scan, averaging over a large number T of independent acquisitions can reduce noise standard deviation by a factor of $T^{-1/2}$. But averaging is not suitable when an underlying dynamic of interest, e.g. bolus passage, is present as the useful features will be washed out much like in a long-exposure photograph of a fast-moving object. Here we show, using Random Matrix Theory (RMT) methods, how to reduce noise in dynamic images, and apply this method to DCE MRI.

2.Methods: Consider T successive images of DCE MR data. While noise in each image is random, the contribution of noise to the spectrum of the covariance matrix of $T >>1$ images is not random and in fact has a distinct universal shape. This shape in the limit of large T is given by Marchenko-Pastur (MP) distribution [1], which is the analog of the Wigner's semicircle law in Wigner-Dyson energy level statistics [2] introduced by the founders of the RMT. This unique shape of the noise in the covariance matrix allows separation of measurement noise and the true physical dynamics of the system. Specifically, we consider DCE MR data: T=120 images of size $N=n\times n$ where N corresponds to the total number of voxels $O(10^4)$. PCA involves forming the sample covariance matrix and finding the S eigenvectors of C corresponding to S largest eigenvalues. S has usually been determined heuristically [3,4,5,6,7]. Remarkably, it can be determined *objectively* based on the fact that the noise eigenvalues gather at the lower limit of the covariance matrix spectrum and follow MP distribution. The eigenvalues outlying the MP distribution represent the significant principal components (PC) which we keep. We first test our methodology on a 100×100 phantom constructed as a combination of ten diagonal small squares each having its own dynamic signal with different SNRs sampled at 1 second intervals for a total of 100 seconds, two off diagonal larger rectangles each with a static signal, background with zero static signal, and identically and independently distributed (IID) $N(0,1)$ noise.

Applications to DCE MRI: GE EPI images of Gd-DTPA administered at a dose of 0.1 mmol/kg and rate of 5mL/s were acquired at 1 second intervals for a total of 60 seconds and 5 second intervals for another 60 seconds totaling 120 samples. Imaging was performed on a 3-T scanner with an 8-channel phased-array head coil. Further imaging parameters are TR 1000 ms, TE 32 ms, 10 contiguous, 3-mm thick axial slices, matrix 128×128, FOV 220×220mm, flip angle 30°, signal bandwidth 1396 Hz/pixel, and in-plane voxel size 1.7×1.7 mm.

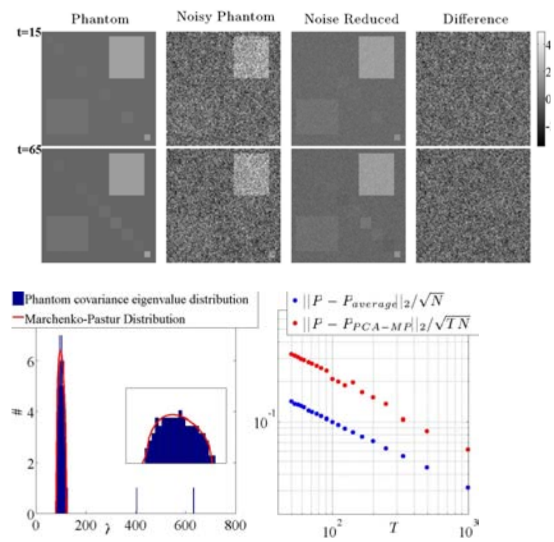

Figure 1: Top: the leftmost and middle left images show snapshots of the phantom, before and after adding noise, at the 15th and 65th acquisitions. The middle right images show snapshots of the noise reduced versions. The rightmost images show the difference between these noisy and reduced noise images. No structural information has been lost. Bottom left: eigenvalue spectrum of sample covariance matrix of the phantom and MP distribution superimposed in red. Four significant PCs are detected. MP estimates variance of noise as 0.99 in agreement with the simulated value. Bottom right: a metric for error, sum over T of squared of difference images divided by TN decreases as $T^{-1/2}$. Red and blue dots correspond to noise reduction by our method for the dynamic phantom and averaging for the static phantom respectively.

3.Results: The spectrum and the superimposed MP distribution for eigenvalues of sample covariance matrix of normal IID noise for the phantom are shown in Figure 1. MP correctly estimates noise variance and distinguishes between noisy and significant eigenvalues. No structural information has been lost according to difference images. Further, this difference decreases as $T^{-1/2}$. Figure 2 shows the eigenvalue spectrum for the sample covariance matrix of DCE MR data. Again, MP achieves a clear separation between noisy and significant PCs. The difference images look mostly random as well.

4.Disussion: Given sufficiently large $T >>1$, the proposed nonlinear noise reduction scheme allows one to increase SNR in all dynamic images simultaneously with the error decreasing as $T^{-1/2}$ similar to the case of averaging over T images in a static case.

5.Conclusion: We presented a nonlinear RMT based noise reduction scheme and examine our method on phantom and DCE MRI.

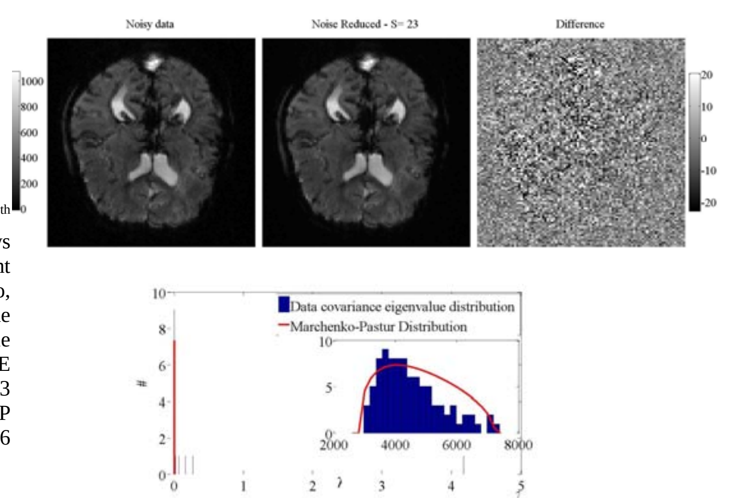

Figure 2: Top: Left image shows the 28th acquisition of the image. Middle image shows the less noisy version considering 23 PCs. Right image shows the difference between these two, which is mostly random indicating no or little structural information loss. Bottom: eigenvalue spectrum of sample covariance matrix of DCE MR data with MP distribution in red. 23 significant eigenvalues are detected. MP estimates standard deviation of noise as 16 compared to that of the background voxels 13.

6.References: **[1]**Marchenko VA, Pastur L. *Matematicheskii Sbornik* 1967;114.4:507-536. **[2]**Dyson FJ. J Math Phys 1962;3:140. **[3]**Torbjørn V, et al. *MICCAI* 2003: 838.**[4]**Eyal E, et al. *J Magn Reson Imaging.* 2009;30(5): 989. **[5]**Balvay D, et al. *Radiology* 2011;258:435-445. **[6]**Qian Z, et al. *Med Image Comput Assist Interv*. 2006;9(1):636. **[7]**Trémoulhéac B, et al. *Structure* 2011;3:5.