A Fast Sampled Projection Method for Assessing Coil Configuration impact on SAR

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Target audience: RF engineers and MR physicists. **Purpose:** We propose a fast method for estimating the fields in a realistic human body model (RHBM) over a wide range of coil configurations by combining the fields computed in the RHBM from a small set of template excitations. The method is effectively an interpolation scheme, and is particularly useful for assessing the sensitivity of local SAR hot-spots to changes in coil design and in coil-body geometry.

Method: One standard numerical approach to computing RF electromagnetic fields in an RHBM, given a set of coils and RF excitation, is to first compute the fields inside a "hollow" head and then compute a RHBM correction. The hollow head field, or the incident field, is just the free-space field and is easily computed from any configuration of coils and excitation, but determining the correction, and hence the total field, requires an expensive electromagnetic analysis. Our strategy is to compute the relationship between incident field and total field for a few template configurations, and to "interpolate" the total field for a new configuration from a few easily computed incident field points. To see this, consider q template incident (hollow-head) electric fields, $\mathbf{E}_1^{\text{inc}}, \mathbf{E}_2^{\text{inc}}, \ldots, \mathbf{E}_q^{\text{inc}}$, in the RHBM, and their corresponding total field solutions, $\mathbf{E}_1^{\text{tot}}, \mathbf{E}_2^{\text{tot}}, \ldots, \mathbf{E}_q^{\text{tot}}$. To obtain an approximate solution for an arbitrary incident field, $\mathbf{E}^{\text{inc}}$, we approximate $\mathbf{E}^{\text{inc}}$ by a weighted combination of the $\mathbf{E}_i^{\text{inc}}$. The weights could be determined by projecting directly on to the template incident fields, at the cost of evaluating the arbitrary incident field at every point inside the RHBM. Instead, the incident field is only evaluated at a set of q sample points, i.e., we evaluate $\mathbf{E}^{\text{inc}}$ at the q points and determine the weights needed to make the approximation an exact equality at the sample points. This requires inverting a q by q interpolation matrix, which is fast as q is assumed small, and moreover, the inverse only needs to be computed once. Thereafter, given the values of $\mathbf{E}^{\text{inc}}$ at the q points, the weights are obtained immediately by multiplying by the inverse matrix. Once the weights are obtained, we exploit linearity and use them to weight the template total fields, $\mathbf{E}_1^{\text{tot}}, \mathbf{E}_2^{\text{tot}}, \ldots, \mathbf{E}_q^{\text{tot}}$, yielding an approximate solution for the incident field $\mathbf{E}^{\text{inc}}$. The entire approximation procedure takes a few seconds at most.

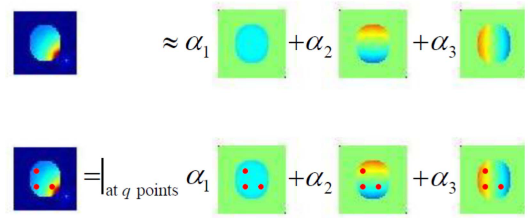

Fig.1. Three incident fields, for which the total fields have been precomputed, are combined to approximate an arbitrary incident field by interpolating at three points.

The error committed in this approximation is a function of both the choice of the basis set $\mathbf{E}_1^{\text{inc}}, \mathbf{E}_2^{\text{inc}}, \ldots, \mathbf{E}_q^{\text{inc}}$, and the choice of interpolation points. For the latter, we use a recently proposed algorithm, known as the Discrete Empirical Interpolation Method [1]. This is a greedy algorithm that runs fast and determines a set of interpolation points guaranteed to yield errors not much higher than that of the best least-squares approximation. For the problem of choosing the basis set, we use an adaptive sampling approach: we start with a small set of $\mathbf{E}_1^{\text{inc}}, \mathbf{E}_2^{\text{inc}}, \ldots, \mathbf{E}_q^{\text{inc}}$. Then, for a given $\mathbf{E}^{\text{inc}}$, we form the interpolation-based approximation, and estimate its error by evaluating $\mathbf{E}^{\text{inc}}$ at some other points, besides the interpolation points. The error in the approximation of $\mathbf{E}^{\text{inc}}$ is a good indicator of the error that will be obtained in the total field solution because the mapping from incident fields to total fields is continuous and well-conditioned [2]. If this error is deemed too large, the solution for $\mathbf{E}^{\text{inc}}$ is computed using the solver and added to the basis set. In this way, the basis set is formed adaptively, and only to the extent necessary to provide a prescribed accuracy.

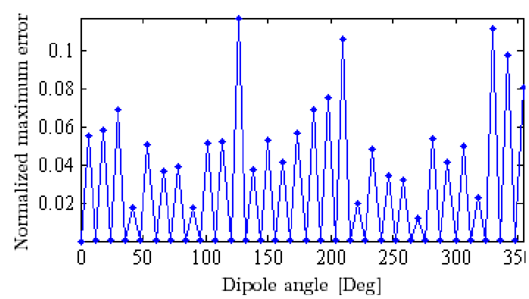

Fig.2. Maximum errors for all three components of the total field, normalized to the maximum value of the total field in the head.

Results: We demonstrate the proposed method with the following simple example. We excite an RHBM (a 10mm resolution DUKE head model) with 30 excitations, each one being a small dipole located on a circle of radius 15.2cm around the head. The dipoles are z-directed and uniformly distributed on the circle, which translates to a spacing of ≈3cm between dipoles. We then use our method to approximate the electric fields that result from exciting with dipoles placed on the same circle, but distributed with half the spacing, so that 30 dipoles coincide with the basis dipoles, and 30 are midway between them. We compute 30 interpolation points in the head, as well as the 30x30 interpolation matrix and its inverse. Then for every one of the 60 positions along the circle, we compute the incident field at the 30 interpolation points, and use these values to generate an approximation for the electric field everywhere in the head. We then compare this approximation with the fields obtained with a full-wave solver (we used an electric field volume integral equation method). This error is shown in Fig. 2, and as can be observed, the approximation drops to zero at the interpolation points, as it should. Between the interpolation points, we usually obtain errors of no more than a few percent, with a few cases in which the errors rise slightly above 10%. Dipole fields at these points could be added to the basis to reduce these errors to zero.

Summary: We described a method for combining electromagnetic field solutions for a set of excitations so as to approximate the solution of nearby excitations. The method is based on interpolating the incident field using a small set of automatically chosen sample points, and yields errors of a few percent in a simple example of dipole excitations around an RHBM. We also described a method for adaptively improving the accuracy by enriching the basis set with excitations for which large interpolation errors are observed.

References: [1] S. Chaturantabut and D. C. Sorensen, *SIAM J. Sci. Comput.*, 32(5), 2737–2764. (2010)
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