Accurate Simulation of Signal and Noise in MRI based on Electromagnetic Field Calculation and Bloch Simulation

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Introduction: Recent years have seen the development of increasingly advanced Bloch solvers approaching more complete MRI system simulators utilizing realistic electromagnetic field distributions and realistic human anatomical models as inputs [1, 2]. For maximum utility, it is important that an MRI system simulator be able to generate realistic noise levels in simulated MR images. Here a method for realistic sample noise generation based on accurately calculated electric fields is presented and validated by comparing the simulated signal-to-noise ratio (SNR) image with that obtained from an actual MR experiment. This work also helps to demonstrate the physical mechanism by which SNR is related to the electromagnetic fields in MRI.

Theory: Simulation of MR images with realistic SNR requires separate generation of accurate MR signal and accurate noise. For accurate MR signal generation, first the equilibrium magnetization vector M_0 is calculated based on classic expression $M_0 = \frac{\rho_0 \gamma^2 \hbar^2}{4 k_B T} B_0$. Here, ρ_0 is the spin density of ^{1}H, γ the gyromagnetic ratio, $\hbar$ the Plank constant, k_B the Boltzmann constant, T the tissue temperature. The evolution of the magnetization vectors of each of the input model voxels due to a specific pulse sequence is then simulated by a series of interplayed rotation matrixes due to effective applied magnetic field B_{eff} and relaxation matrixes due to T_1 and T_2, through time [3]. For accurate MR noise generation, a method is proposed here extending previously-published works. For an N-channel receive array, the NxN noise resistance matrix R is calculated as $R_{ij} = \Delta x \Delta y \Delta z \sum_k (\sigma_k E_{kj} \cdot E_{kj}^*)$ [4], where Δx, Δy, and Δz are the voxel size in x, y, and z directions, σ is the local conductivity, and E_i is the local electric field of receive coil i. Based on the noise matrix, the correlated noise vector for all elements of the receive array for a given ADC event at time t could be simulated as: $S^{corr}(t) = \sqrt{4 k_B T \times \mathrm{BW}}(R^{\frac{1}{2}} * \zeta(t))$ [5], where BW is the ADC bandwidth, and $\zeta(t)$ is a t-specific N-element vector of complex Gaussian random numbers with zero mean and unit variance. To compare the simulated and experimental SNR images, It is expected that the simulated SNR image, acquired by dividing the signal image and noise standard deviation from the above formalism, should match the experimental SNR image, acquired by dividing the signal image and noise standard deviation acquired through separate scans in experiment (with the same scaling from the system receiver chain).

Method: In order to validate the above approach, SNR images acquired with identical sequence protocols from experiment and simulation were compared. The detailed steps required towards this goal are listed as below. ***In experiment***, measurements of the T_1 (104.7 ms, using inverse recovery method), T_2^* (74.8 ms, using multi-echo gradient echo method), conductivity (0.97 S/m, based on probe measurement), and permittivity (82.12, based on probe measurement) were performed on a Siemens quality control phantom (3.75 g $NiSO_4$ x 6 H_20 + 5 g NaCl, per 1000 g distilled H_20). Also, the noise equivalent bandwidth of a 3 T Siemens MRI scanner was measured (0.7813) using a standard method [3]. The phantom was scanned with a extremity birdcage coil (same as simulated) on the real scanner for separate complex signal image and noise image. A 3D non-selective gradient echo sequence (TE = 5 ms, TR = 100 ms, Resolution = 80 by 80 by 60, FOV = 250 by 250 by 300 mm, BW = 40 kHz) was used to ensure a perfectly rectangular slice profile. Gradient and RF spoiling were applied to insure correct measurement of the steady state signal [8]. The experimental SNR image was calculated by dividing the signal magnitude image with the standard deviation of the complex noise image. ***In simulation***, a numerical model of the quality control phantom (using experimentally-measured parameters) and RF coil were created (Fig. 1) and electromagnetic field distributions were calculated using commercially-available software (xFDTD; Remcom Inc.). The digital phantom model with its calculated electromagnetic fields were input into an MRI system simulator [2] and scanned with the same 3D gradient echo sequence for its separate complex signal image and noise image. The simulated SNR image was obtained using the same method as the experimental SNR image. Finally, the SNR images were compared with two extra steps. To ensure the same transmit flip angle, the real phantom was scanned for the experimental B_1+ distribution with the actual flip angle mapping method [7,8], and the simulated B_1+ was scaled so that the value at the center of the phantom was matched to the value in experiment. Also, the bandwidth used to simulate noise was scaled with the experimentally-measured noise equivalent bandwidth (0.7813).

Results and Discussions: The simulated and experimental SNR images are shown in Fig. 2. Although there are some slight differences in the pattern, the absolute values of SNR are very close overall, reflecting the realistic effects of the transmit and receive field distributions of the extremity birdcage coil. The discrepancies in pattern could be due to slight asymmetries in sample placement and coil currents in the experiment. Also, although the maximum SNR value at the center of the simulated SNR image is slightly higher than that from the experiment, this could be explained by the noise figure due to the receiver chain of the real MRI system (<0.5dB) and (again) slight asymmetries in experiment resulting in less-than-perfect constructive interference of RF fields at the center of the coil. Finally, it should be noted that although the proposed method of generating correlated noise for an array was only validated with a single channel birdcage coil, such validation should be equivalent for all diagonal elements for the noise resistance matrix of a multichannel array. Further, it can be deduced that accuracy of diagonal elements ensures accuracy of non-diagonal elements of the noise resistance matrix, since all rely on accurate calculation of the electric field distribution throughout the sample. The above method to generate sample noise can be easily extended to incorporate coil noise for given coil resistance(s), since the coil noise is independent from channel to channel.

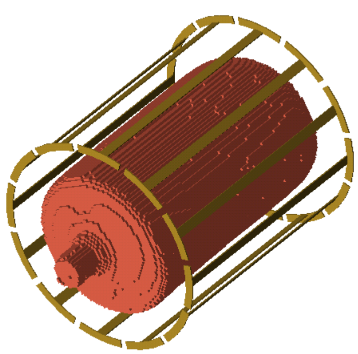

FIG.1. Simulation and experimental setup of a cylindrical phantom inside an extremity birdcage coil.

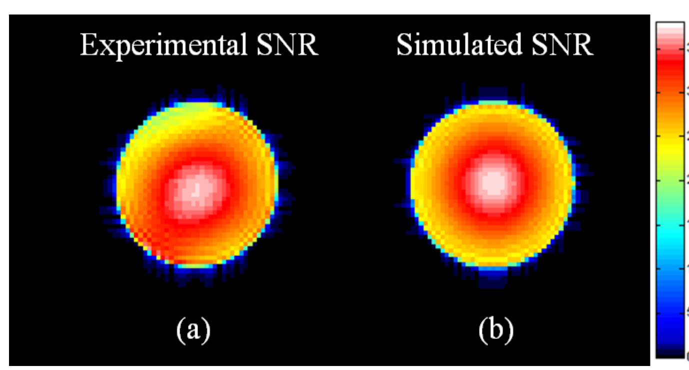

FIG.2. Experimental (a) and simulated (b) SNR images.

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