

Precomputed Green's Functions for Fast Electromagnetic Simulation with Realistic Human Body Models

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Target audience: RF engineers and MR physicists. **Purpose:** We describe a method for accelerating the electromagnetic (EM) simulation of radio-frequency (RF) coils operating in the presence of realistic human body models (RHBM). Such simulations are needed for coil and RF pulse excitation design, specific absorption rate (SAR) prediction, and the design of high-field and parallel-transmission MRI. Simulations obtained by existing state-of-the-art commercial solvers are extremely time-consuming and memory intensive. For example, the EM simulations used in [1] to optimize the RF driving circuitry and excitation took about 2 days with commonly-used commercial EM solvers. In order to enable practical simulation-based optimization of coils, orders of magnitude shorter simulation times are required, while maintaining simulation fidelity. The goal of this study is to speed-up these computations to the point where the evaluation and comparison of thousands of potential coil geometries becomes practical.

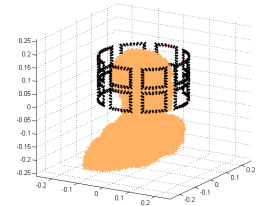


Fig1. 16 Coil 16 Channel Coil Array with 10mm DUKE

Method: In our approach, the electromagnetic effect of the complex RHBM is captured by a *Green's function* (GF), which is simply the electric field due to an elementary current element (a dipole) operating in the presence of the RHBM. The GF is a matrix-valued function that relates the three Cartesian components of the electric field at a point to the three Cartesian components of a source dipole at some other point. The main contribution of this work is the observation that in MRI problems, the current element sources are generally confined to a very small region where the coils are located. The RHBM GF is required only for that specific source region and can therefore be effectively pre-computed and stored for use in a very efficient Method-of-Moment-like (MoM) electromagnetic solver.

Specifically, first the unknown currents on the coils are discretized using piecewise constant basis functions (i.e. small current elements). The total electric field on the coils is expressed as a linear combination of the contributions from each current element using the pre-computed RHBM GF. The currents can then be determined by enforcing a boundary condition at the coils. The main advantage of this MoM with pre-computed RHBM GF is that the number of unknowns (i.e. the number of current segments) is typically around a few hundreds. That is orders of magnitude smaller than the number of unknowns in Finite-Difference or Finite-Element methods, where all the space around and including the RHBM must be discretized. Once our MoM matrix equation is obtained, it can be solved in practically negligible time. As a postprocessing step, the fields can finally be obtained everywhere as a linear combination of RHBM GF fields.

In open space or for simple geometries, such as a conducting sphere, the GF is known analytically and the above approach is straightforward to implement, and it is well-known in the antenna literature. However, for complicated geometries, such as a RHBM, the GF can only be computed numerically, and to our knowledge this has not been attempted in the MRI community. Indeed, a naïve approach of solving for the fields due to dipoles placed on a sufficiently dense grid quickly becomes impractical. However, in this work we show that the required RHBM GF can be further compressed using a small number of basis vectors. Specifically, our offline computation involves 1) obtaining a compressed basis, e.g. by principal component analysis, that accurately approximates any incident field generated by current elements in source region, and 2) assembling a matrix where each column is the solution of a scattering problem in the presence of the RHBM for each vector in the compressed basis. While the solution of the scattering problems in the offline stage is computationally intensive, it is embarrassingly parallelizable, both in shared and distributed memory environments. After precomputing this matrix, we obtain a matrix-based representation of the RHBM GF which can be evaluated fast in the online stage. The online computation involves 1) discretizing the coil conductors into elements, 2) computing the free-space coil-to-coil interaction, using the free-space GF, 3) computing a perturbation of the latter due to the presence of the RHBM, using the RHBM GF, and, 4) summing the contributions from both GFs and solving the resulting linear system. It is important to note that the RHBM GF *only needs to be computed once* for a given RHBM, and it can then be reused for any other coil configuration. Hence, the proposed approach can be exploited in fast coil design optimization flows.

Results and Discussion: As an example, we use a 10-mm DUKE head model (Fig.2(a)) as RHBM at 7T. To solve the scattering problems for the generation of the RHBM GF we use an Electric Field Volume Integral Equation (EFVIE) method. The offline phase takes approx. 55h to generate and solve the 670 vectors in the basis, with a peak memory requirement of 3GB. The storage of the matrices that represent the RHBM GF requires 1GB. We apply the proposed method for solving the port-admittance of a typical 16 channel, 16 coil array (Fig.1). For the given coil configuration, 1440 elements are generated with a 10-mm discretization of the coils, and the port admittance matrix is computed using an iterative approach in approx 9min. Reusing the RHBM GF, for an 8 coil 8 channel configuration, a system of 1104 elements is assembled and solved in approx 4min. For a given current distribution in the 16 coil elements, the proposed approach takes 0.3 sec to compute the fields in the RHBM, whereas the EFVIE takes approx 10 min. The absolute value of the E field for both approaches can be seen in Fig.2(b) and Fig.2(d), whereas the difference between both can be seen in Fig.2(c) (the maximum relative error is less than 0.2%). All approaches were implemented in MATLAB and run on a 24GB RAM Linux server with two E5620 Xeon processors.

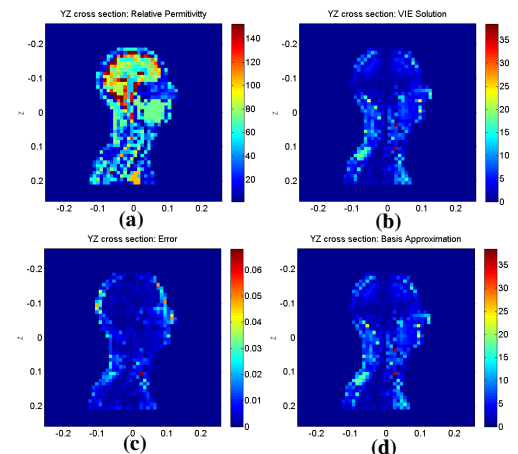


Fig2. 10-mm DUKE model. (a) Relative Permittivity. (b) E field for the EFVIE method. (c) Absolute error in E of proposed approach w.r.t. EFVIE. (d) E field for the proposed approach.

References: [1] M. Kozlov, JMR vol.200 no. 1, pp. 147–152, 2009.

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