Comprehensive Theoretical and Experimental Analysis of the Parametric Framework and SNR of Super-Resolved Spatiotemporally-Encoded (SPEN) MRI

Noam Ben-Eliezer[1], Lucio Frydman[2], and Daniel K. Sodickson[1]
[1]*Bernard and Irene Schwartz Center for Biomedical Imaging, Department of Radiology, New York University School of Medicine, New York, NY, United States,*
[2]*Department of Chemical Physics, Weizmann Institute of Sciences, Rehovot, Israel*

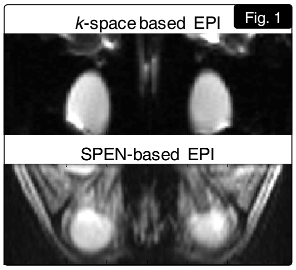

Fig. 1: Human olfactory bulb MRI [Siemens whole-body 3T scanner]

Background The majority of MR imaging is based on encoding the image information in the frequency (***k*-space**) domain. In recent years a conceptually different encoding approach has emerged, based on progressive point-by-point refocusing of the image in the <u>spatial</u> domain through the use of quadratic phase functions [1-4]. This approach, termed Spatiotemporal-Encoding (**SPEN**), provides high robustness against various off-resonance artifacts such as B_0 inhomogeneities and chemical-shift (e.g., fat/water) artifacts [5], and can consequently be used to scan highly challenging regions including the orbitofrontal cortex [6-7], and the olfactory bulb (see Fig. 1, comparing single-shot Echo-Planar images using SPEN vs. *k*-space encoding). In this work we provide a complete parametric framework for the implementation and super-resolved reconstruction [8] of SPEN-based imaging, together with a comprehensive theoretical analysis of its characteristic signal to noise ratio (**SNR**).

Theory SPEN is based on localizing the NMR signal using frequency-swept excitation pulses, which produce a non-linear parabolic phase that acts to dephase all but a single spatial region, located at the parabola vertex. By shifting this vertex across the object, an image can be read without use of a Fourier Transformation (**FT**), but rather using a dedicated, spatially localized, super-resolution (**SR**) reconstruction algorithm. Relying on basic signal-processing principles it can be shown that for a target field-of-view and spatial resolution, a deterministic set of relations govern SPEN's experimental and reconstruction parameters. Two of the key relations are [8]:

$$\textbf{(1)}\quad RF_{TBP} = T_{exc} \cdot BW_{exc} = T_{acq} \cdot BW_{acq} \qquad\qquad \textbf{(2)}\quad F_{SR} = \frac{PSF_{non-SR}}{PSF_{SR}} = \sqrt{N}$$

indicating that **(1)** RF_{TBP}, the excitation pulse time-bandwidth product, can be derived from the acquisition time x bandwidth, and **(2)** that the super-resolution factor F_{SR} (ratio of the experimental and reconstructed point-spread-functions, **PSF**) is equal to $\sqrt{N}$, with *N* denoting the number of acquisition points. Using these relations, SPEN SNR will then compare to that of a *k*-space encoded image via the SR reconstruction matrix $\mathbf{B}$,

$$\textbf{(3)}\qquad SNR^{SPEN} \big/ SNR^{k-enc} = \sqrt{N/(\mathbf{B}^{\dagger}\mathbf{B})}$$

Methods and Results Computer simulations of SPEN PSF based on the newly derived formalism, showed that SPEN and *k*-space encoding produce similar PSFs, suggesting that similar spatial-resolutions can be realized using both methods. Next, a set of numerical calculations and Monte-Carlo simulations was performed to estimate the SNR of SPEN, using a reconstruction model equal to the Hermitian of the encoding matrix $\mathbf{B} = \mathbf{E}^{\dagger}$. Notably, this choice is analogous to conventional FT-based imaging where the encoding and decoding matrices, FT and FT^{-1}, are the inverse of one another. In all cases, predictions resulted in negligible SNR difference between SPEN and *k*-space encoding. Findings were ultimately verified experimentally (Fig. 2) using high-resolution Spin-Echo imaging [Siemens 7T whole-body MRI], and using gold-standard SNR measurements.

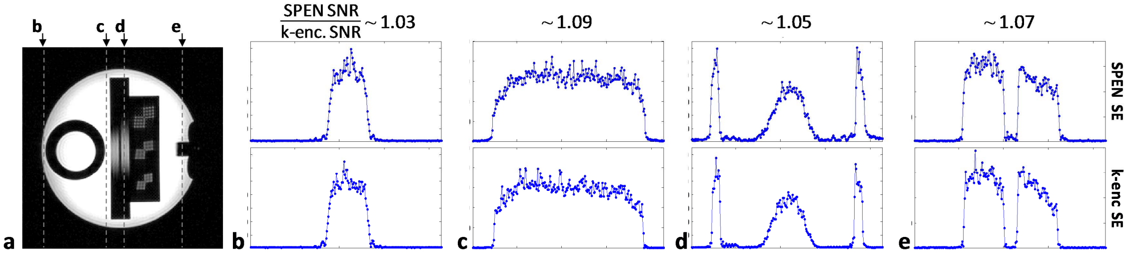

Fig. 2: (**a**) High-resolution image of the phantom used for validating the spatial-resolution equivalence of SPEN and *k*-space encoding. (**b-e**) SNR ratio between SPEN and *k*-space encoding, sampled at four representative 1D profiles along their corresponding 2D SNR maps. Maps were generated using a gold-standard approach where, for each encoding method, a set of 64 images were repeatedly acquired, followed by calculating the ratio between the mean-value and the standard-deviation along the 64-images series, for every spatial-location.

Discussion SPEN has so far been proved highly advantageous for imaging at inhomogeneous fields as well as for chemical-shift imaging applications. Notwithstanding its distinct characteristics, this method is governed by the same fundamental signal-processing principles as *k*-space encoding, resulting in similar averaging properties, and ultimately similar SNR levels as FT based processing. The theoretical analysis presented in this work is applicable to general multidimensional SPEN designs and furthermore provides a unified framework for the analysis of future SPEN and other comparable approaches based on nonlinear phase encoding [9].

References **[1]** Shrot Y, Frydman L. J Magn Reson. 2005, 172(2):179-90. **[2]** Chamberlain R et al. MRM, 2007 58(4):794-9. **[3]** Ben-Eliezer N et al. Magn Reson Imaging. 2010 28(1):77-86. **[4]** Tal A, Frydman L. Prog Nucl Magn Reson Spectrosc. 2010, 57(3):241-92. **[5]** Schmidt R, Frydman L. 2012, MRM [Epub ahead of print] **[6]** Airaksinen AM et al. MRM, 2010 64(4):1191-9. **[7]** Goerke U et al. Neuroimage. 2011, 54(1):350-60. **[8]** Ben-Eliezer N et al. MRM, 2010 63(6):1594-600. **[9]** Witschey WR et al. MRM, 2012 67(6):1620-32.

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