

# A Generalized Series Approach to Sparsely-Sampled fMRI

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## INTRODUCTION

In high resolution functional MRI, it is often desirable to reduce the readout duration to make the acquired data less prone to  $T_2^*$  susceptibility artifacts at the expense of SNR. This can be achieved by undersampling  $k$ -space. However, the conventional Fourier transform-based reconstruction method suffers from undersampling artifacts such as high-frequency ringing and loss of resolution. Several methods have been proposed to address this problem mainly for dynamic or cardiac MRI, including the use of compressed sensing, spatial-spectral support constraint, partial separability, and low-rank constraints [1-3]. This paper proposes a new imaging approach to fMRI with under-sampled data by incorporating the generalized series (GS) constraint in the penalized maximum-likelihood framework.

## THEORY

The acquired fMRI signal is conventionally modeled as

$$d(\mathbf{k}, t) = \int \rho(\mathbf{r}, t) e^{-j2\pi\mathbf{k}\cdot\mathbf{r}} d\mathbf{r} + \xi(\mathbf{r}, t), \quad (1)$$

where  $\rho(\mathbf{r}, t)$  denotes the desired spatial function at particular time point  $t$ , and  $\xi(\mathbf{r}, t)$  is the modeled measurement noise. We assume that the measured data  $d(\mathbf{k}, t)$  is available over a set of points  $\{k_m | k_m \in D, m=1, \dots, M\}$  that sparsely sample  $k$ -space for each time point  $t$ . The undersampling rate is then defined as  $R=N/M$ , where  $N$  is the number of samples in the fully-sampled case. According to the GS model [4], a functional image (in 1D) at a certain time point  $t$  is represented as

$$\rho_{GS}(r, t) = I_{ref}(r) \sum_{l \in \mathcal{J}} c_l(t) e^{-j2\pi l \Delta k r}, \quad (2)$$

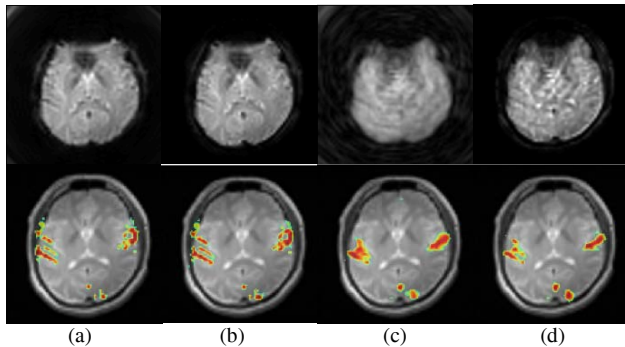
where  $I_{ref}(r)$  is the reference image that contains *a priori* boundary information and helps to incorporate high spatial resolution features,  $c_l(t)$  are unknown GS model coefficients that are functions of time,  $\Delta k$  represents sampling interval satisfying Nyquist criterion and  $l \in \mathcal{J} = \{-L/2, -L/2+1, \dots, L/2-1\}$ . When no nontrivial *a priori* information is available, that is  $I_{ref}(r)=1$ , then (2) reduces to the conventional Fourier series model. While the reference is static,  $c_l(t)$  provides temporal data adaptation, reflecting hemodynamic functional changes over time. Given the signal model (1) and the GS model (2), we then formulate the reconstruction at each time point  $t$  as solving the following penalized maximum-likelihood optimization problem:

$$\hat{\mathbf{c}} = \arg \min_{\mathbf{c}} \|\mathbf{d} - \mathbf{S}\Psi\mathbf{c}\|_2 + \lambda \|\mathbf{c}\|_2,$$

where  $\mathbf{d}$  is the measured data vector,  $\mathbf{S}$  is the linear matrix operator which defines the data measurement and sampling,  $\Psi$  is the matrix operator whose columns are the GS basis functions derived from (2), and  $\lambda$  is the regularization parameter. Having estimated coefficients  $\hat{\mathbf{c}}$ , one can reconstruct the corresponding functional image using (2). The regularization is necessary to stabilize the reconstruction (as opposed to strongly constrain image features as in [5]) because the number of unknown coefficients that carry important dynamic temporal information typically is much higher than the number of measured undersampled data points.

## METHODS & RESULTS

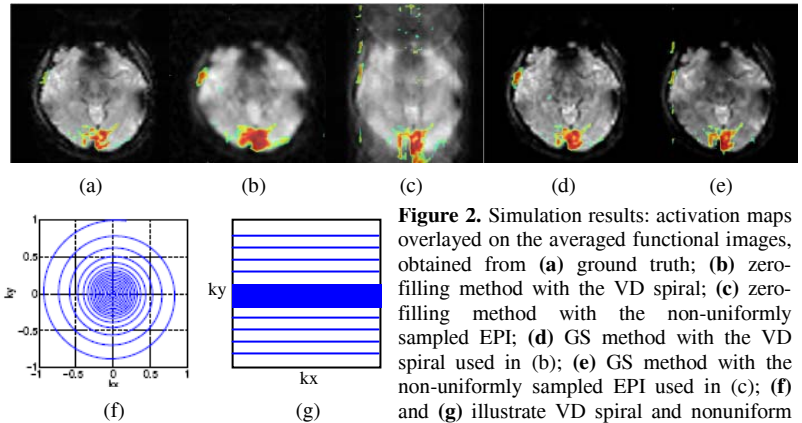
We applied the proposed method to BOLD fMRI data on human volunteers. Data was collected with a single-shot variable-density (VD) spiral-out sequence (64×64 resolution, 30 slices, 128 time frames, FOV = 22 cm, TR=2.04 s, TE=30 ms, 4 mm slice thickness) with full sampling and sparse sampling (undersampling rate  $R=4$ ). A block design consisted of eight “on” and eight “off” blocks, each lasted for 15 sec, resulting in the total duration time of 4 min. On periods consisted of simultaneous visual and auditory stimulation: a circular checkerboard of alternating black and white contrast that reversed at 4 Hz and a randomized tone sequence. Figure 1 shows time-averaged functional images and activation maps, reconstructed with conventional zero-filling and the proposed GS methods. In the full sampling case both methods produced similar reconstructions while in the case of sparse sampling the zero-filling method yielded blurring and artifacts. The GS method suppressed substantial ringing and yielded the activation map with higher spatial resolution. To further validate the method, we also acquire fully sampled fMRI EPI data using the same sensory task. The data was then masked according to two trajectories, that is VD spiral ( $R=3.2$ ) and non-uniform EPI ( $R=3.6$ ) which had 8 center phase encoding lines retained and the rest of the  $k$ -space was sampled by skipping 4 out of 5 lines as illustrated in Figs. 2 (f) and (g). Results confirmed the ability of the method to improve spatial resolution and reduce undersampling artifact such as Nyquist ghost in the case of the non-uniform EPI.



**Figure 1.** Experimental results with undersampling rate  $R = 4$ : averaged functional images (top row) and activation maps (bottom row) from (a) fully sampled spiral data set with the zero-filling method; (b) same as in (a) but with the proposed method; (c) VD spiral data set with the zero-filling method; (d) same as in (c) but with the proposed method.

## REFERENCES

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**Figure 2.** Simulation results: activation maps overlaid on the averaged functional images, obtained from (a) ground truth; (b) zero-filling method with the VD spiral; (c) zero-filling method with the non-uniformly sampled EPI; (d) GS method with the VD spiral used in (b); (e) GS method with the non-uniformly sampled EPI used in (c); (f) and (g) illustrate VD spiral and nonuniform EPI trajectories used in the experiment.

## CONCLUSIONS

A GS approach to sparsely-sampled fMRI data has been presented. The method exploits the GS model in the proposed penalized maximum-likelihood framework to improve spatial resolution of functional images and activation maps and reduce undersampling artifacts. Experimental results demonstrate the effectiveness of the method.

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