Discrete Tomography in MRI: a proof of concept

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TARGET AUDIENCE: Scientists working in the field of MR image reconstruction and processing **PURPOSE:** Segmentation refers to the classification of image pixels into distinct classes, typically based on their grey level. It is usually performed as a post-processing step on an MR magnitude image, which is influenced by reconstruction artefacts. In this abstract, we investigate the integration of reconstruction and segmentation into one single procedure. This combination is a regularized reconstruction problem where we exploit prior knowledge about the discreteness of the grey levels. This research area is known as 'discrete tomography' and has been successfully introduced in CT and electron microscopy. In this abstract, we introduce the concept of discrete tomography to MRI and compare its effectiveness to a standard reconstruction and segmentation technique. **METHODS:** The *Discrete Algebraic Reconstruction Technique* (DART)[1] is an iterative discrete tomography algorithm. It alternates between continuous update steps where the reconstruction is considered as an array of complex-valued unknowns, and discretization steps, which incorporate prior knowledge about the grey values in the image. The reconstruction problem is modeled by the set of linear equations $\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x}$, where $\boldsymbol{y}$ is the k-space data, $\boldsymbol{x}$ is the underlying discrete image and $\boldsymbol{A}$ represents the Fourier encoding matrix. The LSQR[2] algorithm is employed as algebraic reconstruction method (ARM) to solve this linear model. The computationally intensive multiplications with $\boldsymbol{A}$ and $\boldsymbol{A}^*$ are implemented as NUFFT operations on a GPU. Fig. 1 depicts a flowchart of MRI-DART. As a starting point, a first continuous reconstruction $\boldsymbol{x}^{(0)}$ is computed using LSQR. Subsequently, a number of DART iterations is performed, which consist of following steps: **1. Classification + grey level estimation:** We propose a novel thresholding algorithm, which finds the optimal thresholds $\boldsymbol{\tau}_{\mathrm{opt}}$ and grey levels $\boldsymbol{\rho}_{\mathrm{opt}}$ by solving $\operatorname{argmin}_{\boldsymbol{\rho}\in R^{l},\boldsymbol{\tau}\in R^{l-1}} \,||\, \boldsymbol{y} - \boldsymbol{A}\cdot \mathrm{seg}(\boldsymbol{\rho},\boldsymbol{\tau})||_2^2$. **2. Termination criterion:** The algorithm runs for a fixed number of 10 iterations. **3. Identify 'free' and 'fixed' pixels:** In this step, all pixels are classified into one of two categories: fixed or free. The fixed pixels are assumed to have the correct grey level and will not be updated in this iteration. Boundary pixels, for which not all 8 direct neighbour pixels have the same grey level, are added to the set of free unknown pixels. This set is augmented with randomly chosen 15% of the remaining non-boundary pixels in order to allow for changes in other image areas. **4. Apply ARM on free pixels:** Since the number of free pixels is small compared to the total number of pixels, the number of unknown variables in our linear reconstruction problem is vastly reduced, while the number of equations remains the same. This increases the amount of information available to correctly reconstruct the unknown grey levels. **5. Smoothing:** After the reconstruction, a Gaussian smoothing filter with a width of 1 pixel is applied. This compensates for possible erroneously fixed pixels and incorrect grey value estimations. The simulation experiments are based on 2 phantom images: phantom 1 (256x256 pixels) is a binary image which represents blood vessels, while phantom 2 (219x219 pixels) represents a human brain consisting of 4 different grey values. We demonstrate the method here for radial trajectories (varying number of equiangular spokes) and Cartesian trajectories (varying number of phase encoding lines around the center of k-space). We compare the performance of DART to a more conventional approach where the k-space data is first iteratively reconstructed to a 'continuous' image using LSQR, after which a separate segmentation algorithm (Otsu[3]) is employed. **RESULTS:** We consider the reconstruction accuracy of DART as a function of the number of phase encoding lines/spokes. The relative number of misclassified pixels (rNMP) is employed as the accuracy measure, and gives the ratio of the number of misclassified pixels to the total number of pixels. Fig. 2 and 3 compare the reconstruction performances both quantitatively and visually. **DISCUSSION:** The results show that DART yields more accurate reconstructions for both the radial and Cartesian k-space trajectory. This difference is in general more pronounced when the radial sampling strategy is used and when a higher degree of undersampling is employed. **CONCLUSION:** We have introduced the concepts of discrete tomography to the MR imaging model. Simulation results show that the combination of reconstruction and segmentation into one algorithm yields more accurate results than the alternative LSQR+Otsu segmentation technique. Future work will concentrate on expansion of the model by incorporating non-Fourier physical effects which corrupt real MRI data.

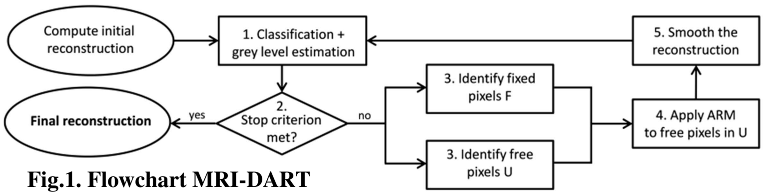

Fig.1. Flowchart MRI-DART

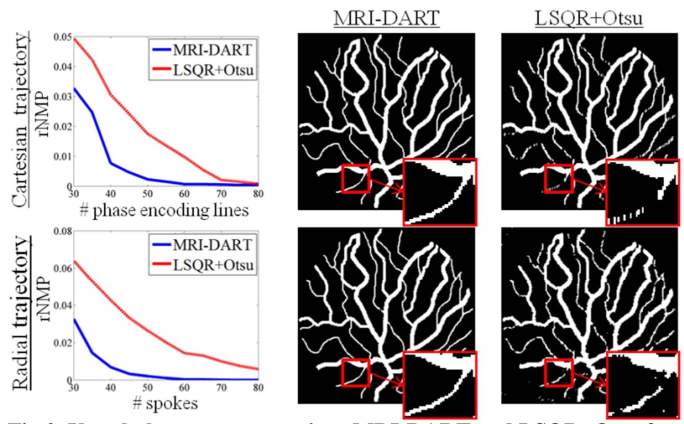

Fig.2. Vessel phantom: comparison MRI-DART and LSQR+Otsu for radial and Cartesian trajectories. Visual results for 50 lines/spokes.

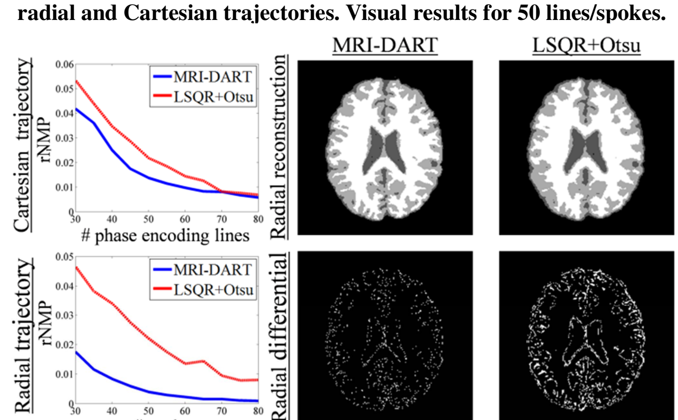

Fig.3. Brain phantom: quantitative comparison MRI-DART and LSQR+Otsu for radial and Cartesian trajectories. Visual results for radial sampling with 30 spokes: comparison of reconstructed images and differential images with ground truth.

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