Edge-Preserving Non-Iterative MAP SENSE MRI Reconstruction

Il Yong Chun[1] and Thomas Talavage[1,2]

[1]*School of Electrical and Computer Engineering, Purdue University, West Lafayette, Indiana, United States,* [2]*Weldon School of Biomedical Engineering, Purdue University, West Lafayette, Indiana, United States*

INTRODUCTION: The Sensitivity Encoding (SENSE) approach to parallel MRI acquisition solves the formulated linear system in the image-domain [1]. As we increase the associated reduction factor (R) and reduce the scanning time, the linear system becomes more ill-posed and the problems of noise amplification and residual aliasing artifact become more serious in the reconstructed image. Although statistical image reconstruction methods can substantially resolve these problems, such methods are iterative and have a high computational cost, particularly when edge-preservation is enhanced through use of appropriate priors [2]. We propose two pre-computation-allowable and non-iterative MAP SENSE reconstruction algorithms based on 1) a Gaussian Random Field (GRF) with non-zero mean and 2) a Huber-Markov Random Field (HMRF) with non-zero mean. Simulation results show that the non-iterative HMRF MAP regularization technique is more effective for edge preservation and residual aliasing artifact reduction than non-iterative GRF MAP and Tikhonov-type regularization methods.

METHOD: The observed reduced-FOV image obtained using R > 1 is treated here as a folded version of the full-FOV image with additive noise: $y = Ax + n$, where y is the observed reduced-FOV image, x is the full-FOV image, n is the noise, and A is the folding matrix as estimated from sensitivity maps. **1) Non-iterative GRF MAP Reconstruction**: We model x by a GRF with non-zero mean x_0 and covariance Ψ^{-1}, i.e. $x \sim N(x_0, \Psi^{-1})$, and assume $n \sim N(0, \Lambda^{-1})$. The MAP reconstruction solution is then $x^* = \underset{x}{\mathrm{argmin}}\{\|x - x_0\|_\Psi^2 + \|y - Ax\|_\Lambda^2\}$ (**Eq 1**). **Eq 1** can have an analytical solution $x^* = x_0 + (A^H \Lambda A + \Psi)^{-1} A^H \Lambda (y - Ax_0)$, where H denotes the transposed complex conjugate. If we pre-compute $G \triangleq (A^H \Lambda A + \Psi)^{-1} A^H \Lambda$ and $g \triangleq (I - GA)x_0$, we may then reconstruct the image by simply computing a matrix-vector product and vector summation: $x^* = Gy + g$. It is reasonable to consider for use as x_0 a low-resolution full-FOV image, such as collected to estimate the sensitivity map, because this is a smoothed version of x. The estimation of the noise covariance matrix Λ^{-1} is based on the assumption that the noise is wide-sense stationary and correlated only over image space and not over k-space. **2) Non-iterative HMRF MAP Reconstruction**: To preserve edges ([3]), we consider a "majorized" HMRF prior ([4]) with non-zero mean x_0 and assume $n \sim N(0, \Lambda^{-1})$. The MAP estimate of x given degraded observation y is then given by the solution to the minimization: $x^{(i+1)} = \underset{x}{\mathrm{argmin}}\left\{\frac{1}{\lambda}\sum_{k=1}^{4}\left(D_k(x - x_0)\right)^t \Gamma_k^{(0)} \left(D_k(x - x_0)\right) + \frac{1}{2}\|y - Ax\|_\Lambda^2\right\}$ (**Eq 2**), where λ is a "temperature" parameter, D_k denotes the convolution operator corresponding to $\sum_{m,n} d_{m,n,k}^t x$, for which $d_{m,n,k}^t x$ are the finite-difference approximations to the 1st-order differences of the image x in four directions: $0°$, $-90°$, $-45°$, and -135 (i.e., $d_{m,n,1}^t x = x_{m,n} - x_{m,n+1}$, $d_{m,n,2}^t x = x_{m,n} - x_{m+1,n+1}$, $d_{m,n,3}^t x = x_{m,n} - x_{m+1,n}$, $d_{m,n,4}^t x = x_{m,n} - x_{m+1,n-1}$), and $\Gamma_k^{(0)} = \mathrm{diag}(\gamma_k)$ with $\gamma_k^{(0)} = \begin{cases} 1 & , |x| \le T \\ T/|x^{(i)}| & , |x| > T \end{cases}$. Because **Eq 2** is a quadratic form, its solution can be computed as $x^* = Hy + h$, where $H \triangleq \left(A^H \Lambda A + \alpha \sum_{k=1}^{4} D_k{}^t \Gamma_k D_k\right)^{-1} A^H \Lambda$, $\alpha = 2/\lambda$, and $h \triangleq \alpha\left(A^H \Lambda A + \alpha \sum_{k=1}^{4} D_k{}^t \Gamma_k D_k\right)^{-1}\left(\sum_{k=1}^{4} D_k{}^t \Gamma_k D_k\right) x_0$. If we pre-compute H and h based on the low-resolution FOV calibration image (as above), reconstruction is performed by simply computing a matrix-vector product and a vector summation.

RESULT: We tested both of the proposed techniques with R=4 and calibration image resolution = 50% of a target image (256x256 simulation data, downloaded from http://www.nmr.mgh.harvard.edu/~fhlin: 3T human MPRAGE data from 8-channel head coil array). Although the RMSE values are similar (**Table 1**), we can observe (**Figs 1-2**) that the HMRF MAP reconstruction algorithm better maintains the energy around edges and reduces residual aliasing artifacts more than GRF MAP reconstruction.

CONCLUSION: We have introduced non-iterative MAP SENSE reconstruction techniques based on GRF and HMRF, using non-zero mean. Our approaches successfully regularize the noise amplification and residual aliasing artifact associated with high reduction factors in SENSE, while preserving more edge energy. In particular, the HMRF MAP technique better preserved edges and reduced aliasing artifacts. These methods promise not only to allow acceleration of MR image reconstruction and permit higher factors of k-space acquisition reduction, but also to deliver more accurately reconstructed images, potentially in real-time.

REFERENCE: [1] K. Pruessmann, et al., Magn Reson Med 1999; 42(5):952. [2] L. Ying, et al., Magn Reson Med 2008; 60(2):414. [3] P. Charbonnier, et al., IEEE Trans on Img Proc 1997; 6(2):298. [4] R. Pan & S. Reeves, IEEE Trans on Img Proc 2006; 15(12):3728.

	GRF MAP	HMRF MAP
RMSE	0.019242	0.020448

Table 1. Error

Non-iter. MAP SENSE using non-zero mean GRF	Non-iter. MAP SENSE using non-zero mean HMRF
$\Psi_{m,n}^{-1} = 1/(1 + \lvert m - n\rvert^{1.8})$	T=0.001 and $\alpha = 1$

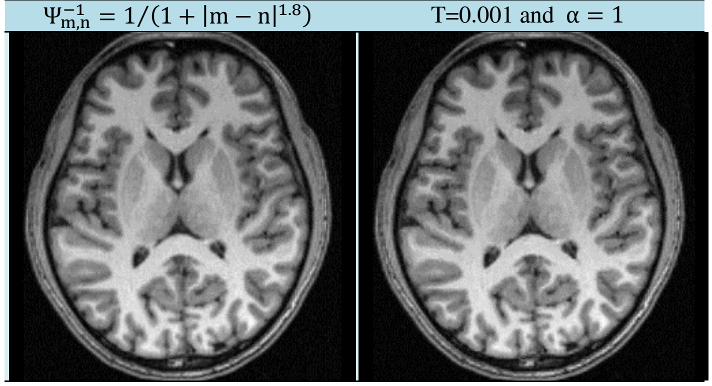

Fig 1. Reconstructed images

Non-iter. GRF MAP	Non-iter. HMRF MAP

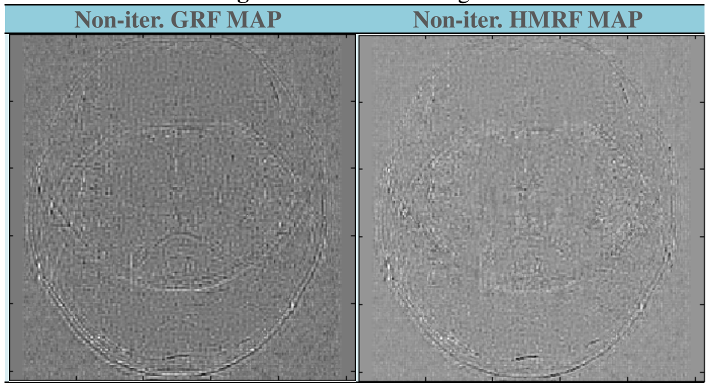

Fig 2. Difference maps with reference image