Elora: Enforcing Low Rank for Parallel MR Reconstruction

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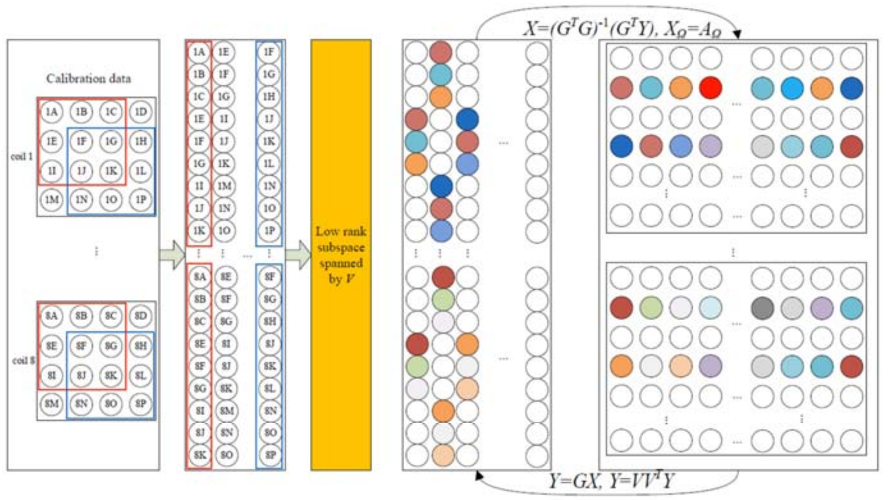
Introduction: Parallel imaging exploits the difference in sensitivities between individual coil elements in a receive array to reduce the number of gradient encodings required for imaging. SENSE [1] and GRAPPA [2] are two representative approaches: GRAPPA utilizes coil sensitivities implicitly via computing the GRAPPA kernel from the calibration data; SENSE explicitly makes use of the coil sensitivities in the problem formulation, and the coil profiles are usually estimated from the calibration data. In this abstract, a new approach, called Elora, is proposed. Elora implicitly uses coil sensitivities by estimating a low-rank subspace from the calibration data, and then works by enforcing the low-rank constraint on the sliding blocks of the *k*-space data.



(a) (b)

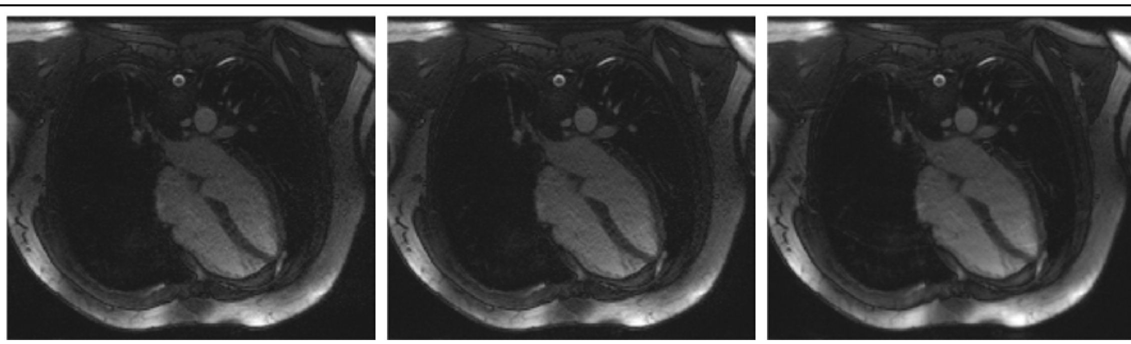
Fig. 1 Illustration of Elora: (a) low rank subspace computation, (b) iterative procedure for optimization. For (a), the first plot illustrates the calibration data for 8 coils; sliding blocks of size 3×3 are generated to form the matrix *C* in the middle plot (e.g., the 3×3 red (blue) block in the left plot is converted to the corresponding 9×1 red (blue) vector in the second plot); and *V*, the low rank approximation of the range space of *C* is computed by SVD in the right plot. For (b), the right plot denotes *X*, which is initialized by the under-sampled *k*-space data (white denotes missing, and the rest are acquired); the left plot denotes *Y*, which is a matrix generated similar way to *C* but on *X* and then projected onto the subspace spanned by *V*, as illustrated by the bottom arrow; and the top arrow illustrates updating *X* given *Y*, i.e., solving *GX*=*Y*, followed by setting the acquired unchanged.

Method: Let n_c denote the number of coils, $c_x \times c_y$ the size of the calibration data, and $k_x \times k_y$ the kernel size. Fig. 1 illustrates Elora. Specifically, Elora firstly forms a calibration matrix *C* by extracting all the sliding $k_x \times k_y$ patches from the calibration (see the middle plot of Fig. 1 (a), and in this example, n_c=8, c_x=c_y=4, k_x=k_y=3), and then computes a low-rank subspace (spanned by *V*) via applying the singular vector decomposition (SVD) to *C* and setting *V* to the leading left singular vectors. A similar idea has been used in [3] for computing the coil profiles. However, to the best of our knowledge, such low-rank subspace has not been directly used in the reconstruction formulation. The low-rank subspace contains the information about the correlations among coils, and VV^TC well approximates *C*, if the rank of *V* is properly set (one way for determining the rank of *V* is to set it as the minimal number that $\| VV^TC\text{-}C\| \leq \alpha \|C\|$, with α being a small number, say 0.05). Elora enforces the low-rank subspace *V* on all the sliding patches of the *k*-space by solving the following optimization problem:

$$\min_{X,Y,X_\Omega=A_\Omega,Y=VV^TY} ||GX-Y||^2 \qquad (1),$$

where Ω denotes the indices of acquired *k*-space data, and A_Ω the acquired data, *X* the *k*-space data of all the coils to be estimated, *G* is an operator that extracts all the sliding $k_x \times k_y$ patches from *X*, and *Y* is the set of sliding patches that lie in the subspace spanned by *V*. Fig. 1 (b) illustrates an iterative procedure for computing *X* and *Y*, which involves matrix-vector multiplication only. Note that, GG^T is a diagonal matrix with positive entries and its inverse can be easily computed.

Results: The cardiac image data was acquired in a healthy volunteer on a 1.5T clinical MR scanner (MAGNETOM Aera, Siemens Healthcare, Erlangen, Germany). Imaging parameters include field of view 370×313 mm^2, flip angle 90°, matrix= 208×176, and 20 coil channels. To obtain the full k-space, segmented acquisition was used. The full k-space was sampled every 4 lines and the center 32 lines were kept for calibration and reconstruction. Fig. 2 shows the reconstruction results by GRAPPA, Elora, and the approach proposed in [4]. GRAPPA and Elora achieved visually comparable results, while the approach proposed in [4] gave reasonably good result. The normalized mean square errors achieved by GRAPPA, Elora, and the approach in [4] are 0.0801, 0.0708, and 0.0888, respectively. The sampling scheme used here might not favor the approach proposed in [4], and thus the results should be interpreted with caution.



(a) (b) (c)

Fig. 2 Reconstructed images: GRAPPA in (a), Elora in (b), the approach of [2] in (c).

Discussion and Conclusion: In this abstract, a new approach, called Elora, was proposed for parallel MR reconstruction. The calibration data were converted to a low-rank subspace in Elora, rather than the kernel vector in GRAPPA. The low-rank subspace might lead to a more robust calibration and thus an improved result. The formulation in (1) can be formulated in the image space so that the image domain structures can be used for reconstruction, and our preliminary result showed further improved results. In comparison with SENSE, the coil profiles were not explicitly estimated so that Elora can avoid the restricted-FOV problem when the pre-specified FOV is smaller than the actual FOV. In comparison with the approach proposed in [4], Elora only needs to perform SVD on a much smaller matrix *C* before the iteration procedure, and no SVD is needed during the iterative procedure while SVD on a much larger matrix is needed for every iteration step of [4].

Disclaimer: The concepts and information presented in this paper are based on research and are not commercially available.

References: [1]. Pruessmann et al., MRM 1999, 42(5): 952-62 [2] Griswold et al., MRM 2002, 47(6): 1202-10 [3] Lustig et al., ISMRM 2011, pp. 479 [4] Lustig et al., ISMRM 2010, pp. 2870.