Quantitative g-factor calculation in k-t-GRAPPA reconstructions

Rebecca Ramb[1], Christian Binter[2], Felix Breuer[3], Gerrit Schultz[1], Maxim Zaitsev[1], Sebastian Kozerke[2,4], and Bernd Jung[1]
[1]Dept. of Radiology, Medical Physics, University Medical Center, Freiburg, Germany, [2]Institute for Biomedical Engineering, University and ETH Zurich, Zurich, Switzerland, [3]MRB Research Center, Würzburg, Germany, [4]Imaging Sciences and Biomedical Engineering, King's College London, London, United Kingdom

Target audience: Researchers working with parallel imaging who are interested in quantitative estimation of the noise enhancement in k-t-GRAPPA reconstructions.

Purpose: Common parallel imaging methods in dynamic MRI, such as k-t-GRAPPA and k-t-SENSE, exploit spatiotemporal dependencies to reconstruct final images resulting in a better, however non-uniform, SNR. A pixel-by-pixel-based quantitative estimation of the non-uniform noise enhancement in parallel imaging was described in literature for GRAPPA (1,3) and SENSE (2), but only for single image acquisitions. In this work, a quantification of the g-factor with regard to temporal frequencies for k-t-GRAPPA reconstructions from time-resolved k-t-accelerated acquisitions is proposed. In k-t-acceleration temporal blurring is observed indicating temporal filtering. The g-factor quantification for k-t-GRAPPA further reflects this temporal filtering, and allows a comparison of performances with other parallel imaging methods in dynamic MRI, or with acceleration methods based on non-time-resolved data. The method is demonstrated in in-vivo data.

$$\mathrm{S}_\alpha = \sum_{\beta=1}^{Nc} \mathrm{S}_\beta^{red} \otimes \omega_{\alpha\beta} \;\; (Nx, Ny, Nt) \quad (1)$$

$$\mathrm{I}_\alpha = \mathrm{FI}_t^{-1}\left\{\sum_{\beta=1}^{Nc} \Omega_{\alpha\beta}\cdot \mathrm{I}_\beta^{red}\right\} \quad (2)$$

$$g_{xf} = \frac{\sqrt{\left|(\mathrm{n}^T\cdot\Omega)\cdot\Sigma\cdot(\mathrm{n}^T\cdot\Omega)^H\right|}}{\sqrt{\left|(\mathrm{n}^T\cdot 1)\cdot\Sigma\cdot(\mathrm{n}^T\cdot 1)^H\right|}} \quad (3)$$

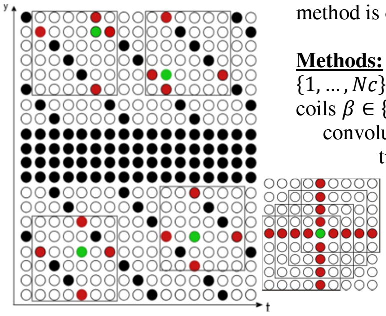

Figure 1: acquisition pattern in ky-t-space for R=5 with highlighted kernel structures (target=green, source=red), from which the convolution kernel is built

Methods: In k-t-GRAPPA, reconstruction of the signal S_α for each coil $\alpha \in \{1,\dots,Nc\}$ is achieved in k-t-space by convolution of the reduced signals S_β^{red} of all coils $\beta \in \{1,\dots,Nc\}$ with afore determined weights $\omega_{\alpha\beta}$ (*Eq. (1)*). According to the convolution theorem, by application of three-dimensional (kx, ky, t) Fourier transforms to reduced signals and weights, reconstruction then becomes a multiplication of the folded images I_β^{red} and weights $\Omega_{\alpha\beta}$, both in x-f-space. The final unfolded image in x-t-space is obtained by Fourier transformation into the time domain (*Eq. (2)*). In order to determine the weights $\Omega_{\alpha\beta}$ in x-f-space, all different kernel structures are combined as illustrated in *Fig. 1* to obtain the convolution kernel, which is flipped in three dimensions, enlarged by zero-filling to fit dimensions (Nx, Ny, Nt) and Fourier transformed in x-f-space. In standard GRAPPA, the weights in image space allow for analytical quantification of the noise enhancement (1). Here, calculation of the combined g-factor in k-t-GRAPPA is achieved in x-f-space by adding temporal frequencies and calculated according to *Eq. (3)*, where $\Sigma := (\sigma^2{}_{\alpha\beta})_{\alpha,\beta\in\{1,\dots,Nc\}}$ denotes the noise covariance matrix and n the vector of coil combination coefficients dependent on the applied combination method. Calculations of g-factors in x-f-space were performed for various acceleration factors and 11 calibration lines retrospectively mimicked from an in-vivo-short-axis CINE data set. Philips Ingenia 3T, 12 channel coil, fully encoded balanced SSFP with 24 time frames and separate scan of coil sensitivities. The analytical quantification was validated using the pseudo-replica-method (4). Pseudo-replica images were obtained by reconstruction in x-f-space of iteratively generated under- as well as fully-sampled data sets with normal distributed noise added in each iteration. Noise enhancement was estimated in a statistical analysis of 265 pseudo-replica images in x-f-space by $\sigma_{red}/(\sigma_{full}\cdot\sqrt{R})$.

Results: *Figure 2(a)* depicts the known attenuation in signal magnitude differences in temporal dimension for higher acceleration factors at a single pixel, which reflects the temporal filtering introduced by temporal interpolation. The pseudo-replica analysis of noise enhancement shows good agreement with the calculated g-factors, as shown in *Fig. 2(b)*. *Figure 3* displays the Roemer combined reconstruction for the first time frame according to *Eq. (2)* with $R = 2$ in *(a)*, together with g-factor maps at $f = 0, 2$ and 4 for $R = 4$ in *(b)* and for $R = 8$ in *(c)*. G-factor values are higher in moving tissue and decrease with respect to higher temporal frequencies, as also visible in the g-factor profiles in *Fig. 3(d)* over all frequencies of the indicated red line.

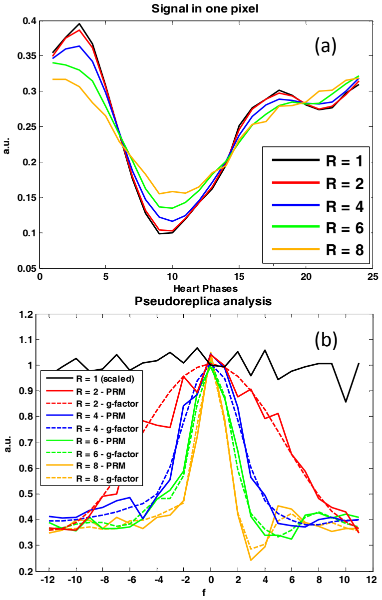

Figure 2: Time course of signal magnitude (a), g-factors and noise enhancement estimation by pseudo-replica-method (PRM) **(b), of the pixel shown in Fig. 3 and for *R=1* scaled by $\sqrt{R}$**

Discussion and Conclusion: The g-factor analysis in x-f-space reflects the noise enhancement performance in k-t-GRAPPA reconstructions in static tissue at $f = 0$ and moving tissue at nonzero frequencies. Values $g_{xf} \leq 1$ are obtained, in contrast to observations in SENSE reconstructions, where $g_x \geq 1$. However, values less than 1 were also observed in standard GRAPPA (4). As recently reported, SNR values in static tissue with k-t-GRAPPA and k-t-SENSE based reconstructions of highly accelerated data acquisition can exceed the SNR of non-accelerated acquisitions (5). The here obtained results reflect the low noise enhancement in k-t-GRAPPA at the expense of temporal filtering in the reconstruction process. Noise enhancement in regions with motion compared to static tissue is moderate for all frequencies. For higher acceleration factors, the g-factor decreases faster for higher frequencies reflecting the effect of low-pass filtering, but remains at the same level for static tissue. General noise enhancement with respect to frequencies in k-t-GRAPPA is smooth, similar to the observation of smooth g-factor variations in the spatial domain for standard GRAPPA. The method provides a measure for the noise enhancement and temporal filtering in k-t-GRAPPA reconstructions.

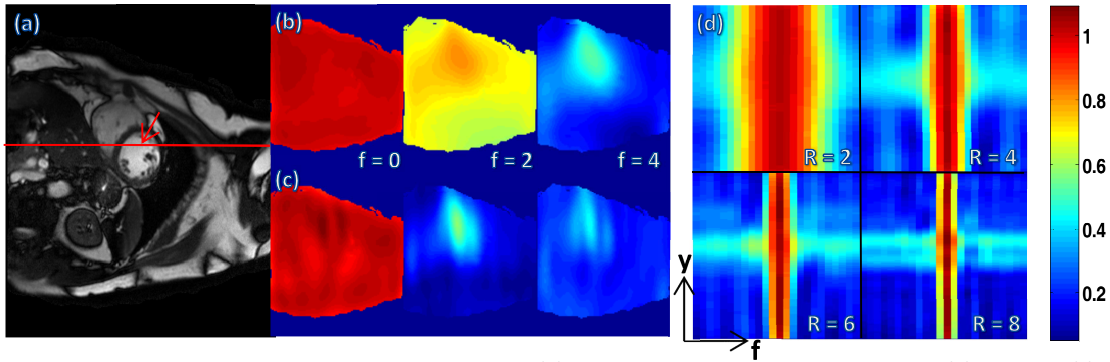

Figure 3: Reconstructed image at time frame = 1 for R = 2 (a), g-factor maps at f = 0, 2 and 4 for R = 4 (b) and R = 8 (c), g-factor of the red line over all frequencies for R = 2, 4, 6 and 8 (d)

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