Optimal reconstruction method of SENSE imaging depending on the desired voxel function

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Purpose: In the generalized theory of SENSE imaging, the reconstruction is described by a linear transformation of the (undersampled) k-space data into image domain allowing a free choice of a set of desired voxel functions (DVF) $i_\rho(r)$ (i.e. point spread functions for the voxel at position ρ) [1]. In the strong voxel approach (SVA) the reconstruction is defined by a least squares minimization of the resulting voxel functions (RVF) $f_\rho(r)$ to the DVF $\|f_\rho(r)-i_\rho(r)\|$. On the other hand, in the weak voxel approach (WVA), the set of RVF fulfil the orthonormality relation to the set of desired voxel functions: $\int i^*_\rho(r) f_{\rho'}(r)dr=\delta_{\rho,\rho'}$. Pruessman et al. [1] presented a solution of the reconstruction matrix for the WVA with Dirac distributions as DVF, i.e. the voxel function is one in the voxel centre and zero outside of the voxel centre. Last year, a generalized solution of SENSE imaging for the WVA and SVA was presented which allowed choosing DVFs differing from Dirac distributions [2]. The purpose of this work was to examine the resulting voxel functions depending on the choice of the desired voxel functions for the weak and the strong voxel approach.

Methods: Identical notations as in [1] are used in this work. Approximating the coil sensitivities with polynomials $s_\gamma(x)=\sum_{l=0}^{M} c_{\gamma,l}(x-x_0)^l$ of order M, an analytic expression of the matrix elements of the encoding matrix $E_{(\gamma,\kappa),\rho}$ and of the correlation matrix of the encoding functions $C_{(\gamma,\kappa)(\gamma',\kappa')}$ can be derived: $E_{(\gamma,\kappa),\rho}=\sum_{l=0}^{M} c_{\gamma,l}(-i)^l \frac{d^l}{dk^l}(i_{\rho=0}(k)e^{ikx})\big|_{k=k_\kappa}$; $C_{(\gamma,\kappa)(\gamma',\kappa')}=\sum_{l=0}^{M} c_{\gamma,l}(-i)^l N \frac{d^l}{dk^l}(\mathrm{sinc}(0.5Nk)e^{-0.5ik})\big|_{k=k_\kappa-k_{\kappa'}}$, with γ: index of the coil channels, κ: sampled k-space position, $i_{\rho=0}(k)$: Fourier transform of the desired voxel function [2]. The reconstruction matrices of WVA and SVA are defined as $F_{\mathrm{WVA}}=(E^H\Psi^{-1}E)^{-1}E^H\Psi^{-1}$ and $F_{\mathrm{SVA}}=E^HC^{-1}$ (Ψ: sample noise matrix) [1]. A study of a human head was performed on a 1.5T scanner with spin-echo imaging (T_R=2000 ms, T_E=10 ms, flip angle 90°, slice thickness 5 mm, matrix 256x256, FOV 260 x260 mm²) and a 4 channel receiver coil. The evaluated DVF were sinc and Gaussian shaped and the WVA and SVA were applied to reconstruct the Nyquist sampled data set. The resulting voxel functions (RVF) were calculated with: $f_\rho(r)=\sum_{\rho,\kappa} F_{\rho,(\gamma,\kappa)}s_\gamma(r)e^{ik_\kappa r}$.

Results: For the sinc shaped desired voxel function the WVA and SVA provided similar reconstruction accuracy (fig 1a/1c and 1b/1d). This is confirmed by the resulting voxel functions (fig 2a) which are both identical to the DVF. In case of a Gaussian shaped desired voxel function the image of the WVA shows extensive Gibbs ringing (fig 1e/1g). In contrast, the image reconstructed using the SVA (Figure 1f/1h) is free of artifacts. While the resulting voxel function of the SVA is similar to the desired voxel function, the RVF of the WVA has a smaller mainlobe and massive sidelobes (fig 2b). The calculated product of DVF and RVF ($\mathrm{DVF{*}RVF}=\int i^*_\rho(r)f_{\rho'}(r)dr=\delta_{\rho,\rho'}$) for the 16 nearest voxel functions can be seen for the WVA and SVA in Figure 2c and 2d. For the sinc shaped desired voxel function, both the WVA and SVA fulfill the orthonormality relation (i.e. Dirac distribution at r=0) (fig 2c). Contrary, only the resulting voxel functions of the WVA are orthonormal to the Gaussian shaped DVF (fig 2d).

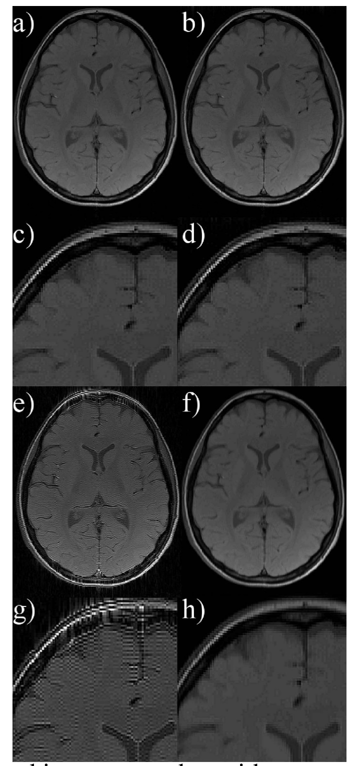

Figure 1: Reconstructed images together with a magnified region for sinc shaped DVF using the WVA (a, c) and the SVA (b, d) and for Gaussian shaped DVF using the WVA (e, g) and SVA (f, h).

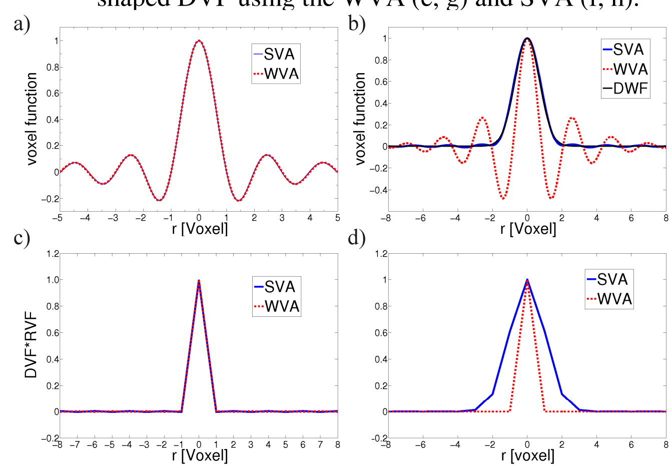

Figure 2: RVF (a, b) and the product DVF * RVF (c, d) of the sinc (a, c) and Gaussian shaped (b, d) DVF.

Discussion: The SVA of SENSE imaging is more robust in the reproduction of the desired voxel functions because they are calculated by a least squares minimization (fig 1). The resulting voxel functions of the WVA are orthonormal to the desired voxel functions (fig 2). Therefore, only in case of an orthonormal set of desired voxel functions (for example a set of sinc funtions) the WVA provides the identical resulting voxel functions to the DVF. In all other cases (including a Gaussian shaped DVF), the resulting voxel functions are complex oscillating functions which maximize the overlap with the DVF at the same position and minimize the intergral of the overlap with all other neighboring desired voxel functions.

Conclusion: The new algorithm presented in [2] improves the reconstruction of SENSE imaging because it allows applying more realistic desired voxel functions than the Dirac distributions used in [1]. The SVA is the more robust reconstruction method as it reproduces the desired voxel function well. The WVA is a reliable and numerically advantageous method for orthonormal DVFs.

References: [1] Pruessmann KP, et al. Magn Reson Med 1999, 42: 952. [2] Gutberlet M. Proceedings ISMRM 2012, 2221.