Efficient non-Cartesian SPIRiT without explicit consecutive regridding and gridding

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Introduction: SPIRiT [1] is an autocalibrating parallel imaging (PI) method for arbitrary k-space trajectories generalizing GRAPPA [2]. Recently, the computational complexity of the calibration operator has been reduced from $O(N_c^2)$ to $O(N_c)$ (N_c: Number of coils) by extracting coil sensitivity maps via eigen decomposition of the interpolation kernel [3,4]. In [3] only the forward operation of the modified kernel has been used in a projection over convex sets (POCS) algorithm for Cartesian sampling. However, conjugate gradient (CG) type solvers for non-Cartesian sampling also include adjoint besides forward operations. In this work, we reduce the computational complexity for non-Cartesian SPIRiT by incorporating the coil sensitivity-based kernel into CG-like reconstruction. Additionally, the two consecutive k-space interpolation steps during the regridding-gridding operation are approximated by a diagonal matrix multiplication potentially reducing computational costs further.

Theory: In SPIRiT, the GRAPPA-like k-space interpolation kernel can efficiently be implemented in image domain yielding a matrix-vector multiplication, $\boldsymbol{\rho}_n(\mathbf{x}) = G(\mathbf{x})\, \boldsymbol{\rho}_{n-1}(\mathbf{x})$, for each pixel at iteration step n. $\boldsymbol{\rho}_n(\mathbf{x})$ denotes the (N_cx1) column vector stacking each coil's image value at pixel position $\mathbf{x}$ and $G(\mathbf{x})$ is a (N_cxN_c) matrix containing the values of the Fourier transformed k-space kernel at $\mathbf{x}$. The corresponding image domain operator acting on all coil images stacked in vector $\boldsymbol{\rho}$ is then denoted as $\mathbf{G}$. It has been shown in [3] that the pixel-wise $O(N_c^2)$ matrix multiplication can be reduced to a successive $O(N_c)$ vector-vector and scalar-vector multiplication: $G(\mathbf{x}) \approx \|c(\mathbf{x})\|^{-2} c(\mathbf{x}) c(\mathbf{x})^H$, where $c(\mathbf{x})$ is the coil sensitivity vector corresponding to the eigenvector of $G(\mathbf{x})$ with eigenvalue 1. Defining the modified operator as $C(\mathbf{x}) = \|c(\mathbf{x})\|^{-2} c(\mathbf{x}) c(\mathbf{x})^H$ and $\mathbf{C}$, respectively, we see that $C(\mathbf{x})^H = C(\mathbf{x})$ and $\mathbf{C}^H = \mathbf{C}$. With the identity $\mathbf{I}$, the calibration consistency and its adjoint operation appearing as $(\mathbf{G} - \mathbf{I})^H (\mathbf{G} - \mathbf{I})$ in CG-like reconstruction can then be simplified to $(\mathbf{C} - \mathbf{I})^H (\mathbf{C} - \mathbf{I}) = -(\mathbf{C} - \mathbf{I})$.

Following [5] we replace the regridding-gridding operation $\mathbf{E}^H\mathbf{E}$ with the encoding matrix $\mathbf{E}$ of the data consistency term with $\mathbf{I}_{zp}{}^H\mathbf{F}^H diag(\mathbf{F}_0\mathbf{Q})\mathbf{F}\mathbf{I}_{zp}$ [6], with the zero-padding matrix $\mathbf{I}_{zp}$ doubling the image matrix size, the unitary discrete Fourier transform (DFT) $\mathbf{F}$, the unnormalized DFT $\mathbf{F}_0$, and $\mathbf{Q}$ as defined in [5]. Instead of calculating $\mathbf{Q}$ according to [5], we approximate $diag(\mathbf{F}_0\mathbf{Q})$ by another diagonal matrix $\mathbf{K}$. Similar to [7], $\mathbf{K}$ is obtained by regridding a constant ones k-space onto the non-Cartesian trajectory followed by gridding back onto the Cartesian grid. Combining the two above approaches, the normal ecquation to solve the SPIRiT image domain minimization problem for $\boldsymbol{\rho}$, $argmin_{\boldsymbol{\rho}}\, \|[\mathbf{E}^H\mathbf{E} + \lambda^2(\mathbf{G} - \mathbf{I})^H(\mathbf{G} - \mathbf{I})]\boldsymbol{\rho} - \mathbf{E}^H\mathbf{d}\|^2$, with CG reduces then to: $argmin_{\boldsymbol{\rho}}\, \|[\mathbf{I}_{zp}{}^H\mathbf{F}^H\mathbf{K}\mathbf{F}\mathbf{I}_{zp} - \lambda^2(\mathbf{C} - \mathbf{I})]\boldsymbol{\rho} - \mathbf{E}^H\mathbf{d}\|^2$, with the arbitrary k-space trajectory $\mathbf{d}$.

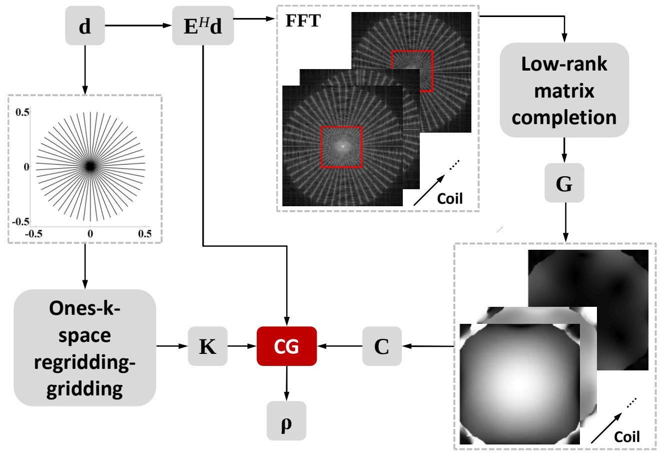

Figure 1 Reconstruction workflow. $\mathbf{K}$ is obtained via the geometry of the undersampled trajectory. The calibration operator $\mathbf{G}$ and $\mathbf{C}$ are calculated from the center of k-space via low-rank matrix completion and eigen decomposition, respectively.

Methods: An artificial 16-channel coil array data set [8] was used to generate a reference multi-coil computer model data set. Complex valued white Gaussian noise with independent real and imaginary part was added. 8 virtual channels were then computed using coil array compression [8]. The reference data (256x256 matrix) was projected onto undersampled spiral and radial k-space trajectories. For both sampling schemes, a fully sampled k-space center (30x30) for calibrating $\mathbf{G}$ with a (7x7)-kernel was calculated via low-rank matrix completion [9]. $\mathbf{C}$ was obtained by eigen decomposition of $\mathbf{G}$ and $\mathbf{K}$ via regridding and gridding of a (256x256)-ones-k-space (Fig.1). CG with 40 iteration steps was then used for reconstruction, once with $\mathbf{G}$ and $\mathbf{E}$ for standard SPIRiT, once with both new operators $\mathbf{C}$ and $\mathbf{K}$, and once with $\mathbf{C}$ and $\mathbf{E}$. To implement $\mathbf{E}$, the NUFFT gridder [10] was used.

Results: Fig. 2 shows reference, direct non-uniform Fourier transformed, SPIRiT reconstructed and error images for the simulated spiral and radial data. The masked error images depict the equality of using operator $\mathbf{G}$ or $\mathbf{C}$ and $\mathbf{E}$ or $\mathbf{K}$. Compared to standard spiral SPIRiT with $\mathbf{G}$ and $\mathbf{E}$ the saving in reconstruction time when using $\mathbf{C}$ and $\mathbf{K}$ was 43% and 40% with $\mathbf{C}$ and NUFFT gridder $\mathbf{E}$. For the radials, the corresponding time savings were 35% and 49% revealing that the benefit of $\mathbf{K}$ depends on the number of acquired k-space samples.

Discussion: A modified coil-sensitivity based calibration operator was incorporated into non-Cartesian CG-like SPIRiT. While maintaining image quality, significant reduction in reconstruction time has been demonstrated for simulated spiral and radial data. In addition, the exchangeability of the two consecutive k-space interpolation steps with a diagonal matrix multiplication has been shown. Depending on the number of k-space samples, reconstruction times on the order of the highly optimized NUFFT gridder are achieved.

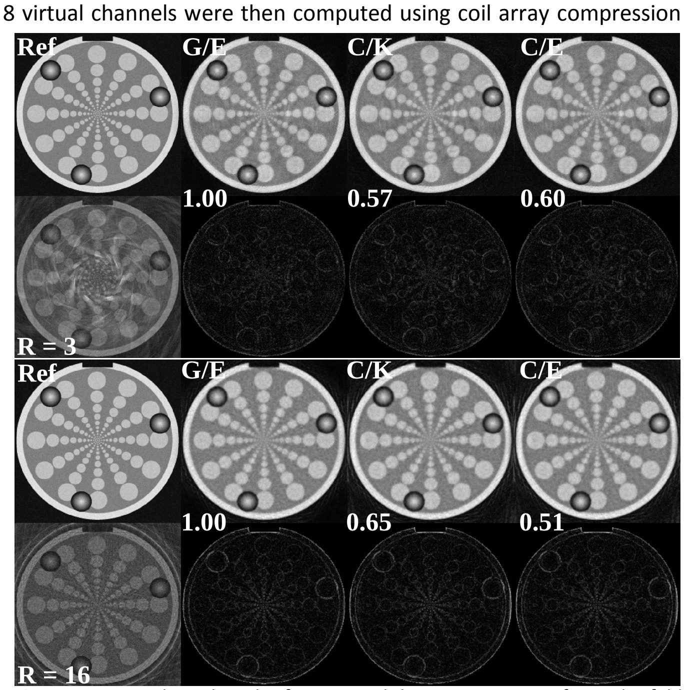

Figure 2 Top: Coil-combined reference and direct Fourier transformed 3-fold undersampled spirals. Reconstructed and error images are shown for standard SPIRiT with operators $\mathbf{G}$ and $\mathbf{E}$ and for the proposed method using operators $\mathbf{C}$ and $\mathbf{K}$ or $\mathbf{C}$ and $\mathbf{E}$. Reconstruction times relative to standard SPIRiT (1.0) are also depicted. Bottom: Corresponding illustrations for radial data set (R=16).

References: [1] Lustig M, MRM (64) 2010, [2] Griswold MA, MRM (47) 2002, [3] Lai P, ISMRM 2010:345, [4] Lustig M, ISMRM 2011:479, [5] Wajer FTA, ISMRM 2001:767, [6] In personal communication with Matthias Seeger (EPFL), [7] Akcakaya M, ISMRM 2011:2550, [8] Buehrer M, MRM (57) 2007, [9] Lustig M, ISMRM 2011: 483, [10] Fessler JA, IEEE TSP (51) 2003.