A Dictionary-based Graph Cut Algorithm for MRI Reconstruction

Jiexun Xu[1], Nicolas Pannetier[2], and Ashish Raj[3]

[1]*Department of Computer Science, Cornell University, Ithaca, New York, United States,* [2]*Department of Radiology and Department of Veterans Affairs Medical Center, University of California at San Francisco, San Francisco, CA, United States,* [3]*Department of Radiology, Weill Medical College of Cornell University, New York, NY, United States*

INTRODUCTION

Among recent parallel MR imaging reconstruction methods, a Bayesian method called Edge-preserving Parallel Imaging with GRAph cut Minimization (EPIGRAM[1]) proposes highly non-convex Edge Preserving Priors (EPP) that have proven successful in lower level vision, statsitics, and image processing. On the other hand, recent Compressive Sensing (CS) methods[2] employ random-undersampling schemes that produces incoherent, noise-like aliasing artifacts, which are easy to remove by imposing L1 norm priors. Both methods have demonstrated improved visual quality over regularized SENSE[3]. Unforunately, EPIGRAM is only computationally tractable on Cartesian under-sampled data, while CS is usually restricted to convex L1 norms and cannot employ more sophisticated but desirable EPP priors as these priors make the problem highly non-convex and difficult to solve. We combine the advantages of both methods, and propose an algorithm that iteratively solves the resulting optimizating problem inside a well-constructed dictinary that reduces the dimensionality of the problem. We apply graph cuts[4], a well-established binary optimization routine, to a binary optimization problem in each iteration inside a propery constructed dictionary. We solve the resulting optimization problem. Preliminary *in vivo* validation for 3D brain MPRAGE data at 7x simulated accelerations is reported. This work represents one of the first MR reconstruction algorithms that can handle truly non-convex sparsity inducing priors.

METHOD

Parallel imaging is converted to a Bayesian energy minimization problem: $\|E\mathbf{x} - \mathbf{y}\|^2 + G_{EP}(\mathbf{x})$, where $\mathbf{x}$ is the vectorized set of desired image intensities, G_{EP} is an EPP prior that encourages edge preserving piecewise smoothness, $\mathbf{y}$ are coil outputs and matrix E captures the sensitivities of different coils.

To combine the advantages of CS and EPIGRAM, we present a new objective: $\widehat{\boldsymbol{\alpha}} = \arg\min_{\boldsymbol{\alpha}} \|E(\mathbf{x}^0 + V_0\boldsymbol{\alpha}) - \mathbf{y}\|^2 + G_{EP}\left((\mathbf{x}^0 + V_0\boldsymbol{\alpha})\right)$, and the final image is $\hat{\mathbf{x}} = \mathbf{x}^0 + V_0\widehat{\boldsymbol{\alpha}}$. The dictionary V_0, consists of high frequency (edge) information derived from an initial solution $\mathbf{x}^0$, which in our case is the SENSE image. The dictionary contains far fewer members than the image size. So the dimensionality of the problem is greatly reduced, which allows graph cut techniques to be applied without incurring prohibitive cost. Unlike traditional dictionary based methods that rely on pre-existing training images, we rely only on $\mathbf{x}^0$ as inappropriately recruited features from external datasets might be risky in medical imaging.

We minimize the new objective iteratively by iteratively solving a binary minimization problem with graph cuts: $\widehat{\boldsymbol{b}}^k = \arg\min_{\boldsymbol{b}} \|E(\mathbf{x}^{k-1} + V^k\boldsymbol{b}) - \boldsymbol{y}\|^2 + G_{EP}\left((\mathbf{x}^{k-1} + V^k\boldsymbol{b})\right)$, where $\mathbf{x}^{k-1}$ is the solution after last iteration, V^k is a matrix generated from current iteration with a heuristic, and $\boldsymbol{b}$ is the binary variable to be minimized. We update the solution as $\mathbf{x}^k = \mathbf{x}^{k-1} + V^k\widehat{\boldsymbol{b}}^k$.

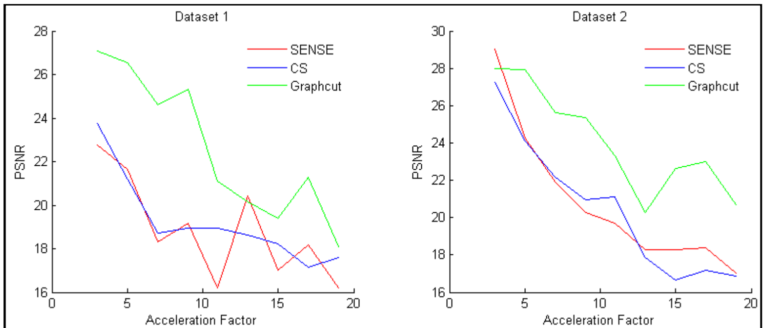

Fig 1 PSNR scores (y-axis) of SENSE, CS and Graphcut results at various acceleration factors (x-axis) on two datasets

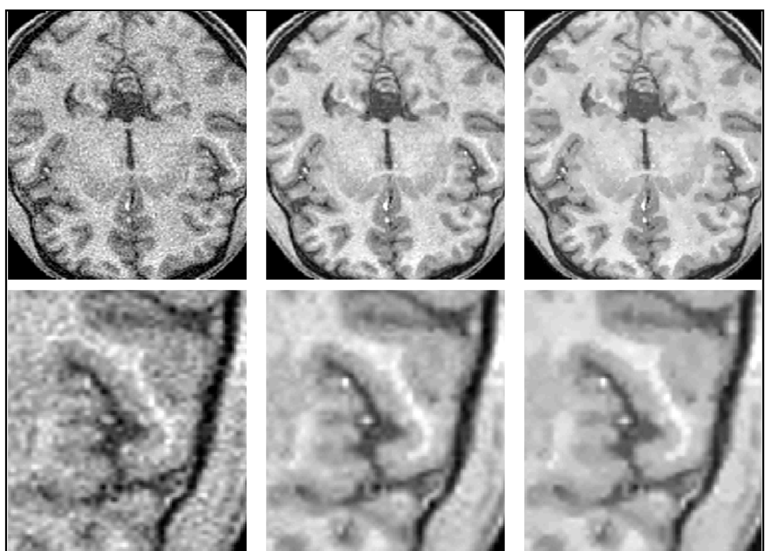

Fig 2 Reconstruction results for one brain slice: SENSE (top left), CS (top middle), and our result (top right). The bottom row is a zoomed view.

RESULTS

T_1 weighted MPRAGE 3D brain data was acquired at 3T on a Siemens Skyra scanner with 16 channels head matrix coil. We sampled the full data and reconstruction results with both CS and our algorithm. We simulated accelerations from 3x to 19x and reconstructed the images with SENSE, CS and our method (Graphcut). Each reconstruction takes between 4~6 hours. We computed the PSNR scores of these results on two datasets and plot the results in Fig 1. In both datasets, our method results in generally higher scores compared to the other two methods at various acceleration factors. As quantitative measurements are imperfect for describing an image, we visually demonstrate the recon results in Fig 2 at 7x on another dataset. Due to the power of the EPP model our algorithm used, our result removes more noise and aliasing artifacts than the other two methods, and produces stronger edges and sharper boundary between gray matter and white matter.

CONCLUSION

Our algorithm successfully combines the advantages of EPIGRAM (EPP prior) and CS (random under-sampling) via applying graph cuts inside a dictionary. Our algorithm has produced higher quality images than SENSE and CS with various *in vivo* data at high acceleration. We believe that our algorithm has demonstrated good promise, and future work such as identifying better dictionary vectors and EPP models can result in even better images, while parallelization efforts can improve runtime and achieve clinical feasibility.

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