

# Voxel function and signal-to-noise ratio (SNR): What are the optimal reconstruction method and sampling strategy in SENSE imaging?

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**Purpose:** In the generalized theory of SENSE imaging, the reconstruction is described by a linear transformation of the (undersampled) k-space data into image domain and by choosing a set of desired voxel functions (DVF)  $i_{\rho}(r)$  (i.e. point spread functions for the voxel at position  $\rho$ ) [1]. In the strong voxel approach (SVA) the reconstruction is defined by a least squares minimization of the resulting voxel functions (RVF)  $f_{\rho}(r)$  to the DVF  $\|f_{\rho}(r) - i_{\rho}(r)\|$ . Alternatively, in the weak voxel approach (WVA) the set of RVF fulfil the orthonormality relation of the set of desired voxel functions:  $\int i_{\rho}(r) f_{\rho'}(r) dr = \delta_{\rho, \rho'}$ . Pruessman et al. [1] presented a solution of the reconstruction matrix for the WVA with Dirac distributions as DVF. Last year, a generalized solution of SENSE imaging for the WVA and SVA was presented which allowed choosing DVF differing from the Dirac distributions [2]. Purpose of this work was to examine the SNR efficiency of the WVA and SVA depending on the desired voxel functions.

**Methods:** Identical notations to the SENSE formalism are used as in [1]. Approximating the coil sensitivities with polynomials  $s_{\gamma}(x) = \sum_{l=0}^M c_{\gamma,l} (x - x_0)^l$  of order  $M$ , an analytic expression of the matrix elements of the encoding matrix  $E_{(\gamma, \kappa), \rho}$  and of the correlation matrix

of the encoding functions  $C_{(\gamma, \kappa)(\gamma', \kappa')}$  can be derived:  $E_{(\gamma, \kappa), \rho} = \sum_{l=0}^M c_{\gamma,l} (-i)^l \frac{d^l}{dk^l} (i_{\rho=0}(k) e^{ikx}) \Big|_{k=k_{\kappa}}$ ;  $C_{(\gamma, \kappa)(\gamma', \kappa')} = \sum_{l=0}^M c_{\gamma,l} (-i)^l N \frac{d^l}{dk^l} (\text{sinc}(0.5Nk) e^{-0.5ik}) \Big|_{k=k_{\kappa} - k_{\kappa'}}$ , with  $\gamma$ :

index of the coil channels,  $\kappa$ : sampled k-space position,  $i_{\rho=0}(k)$ : Fourier transform of the desired voxel function. The reconstruction matrices of WVA and SVA are defined as  $F_{WVA} = (E^H \Psi^{-1} E)^{-1} E^H \Psi^{-1}$  and  $F_{SVA} = E^H C^{-1}$  ( $\Psi$ : sample noise matrix).

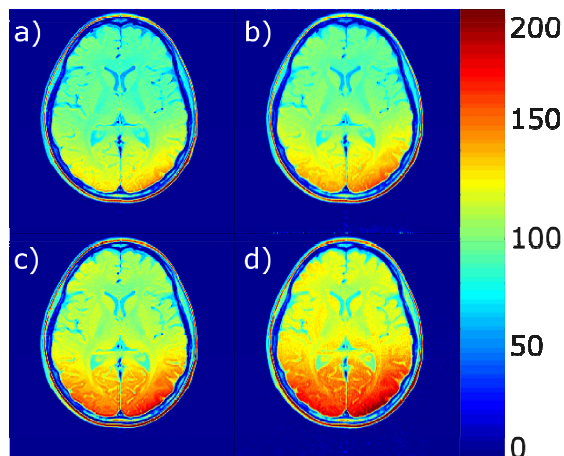
Because the Hanning window is a common k-space filter to reduce Gibbs artifacts, the Fourier transforms of the ideal voxel functions were chosen to be Hanning shaped  $i_{\rho=0}(k) = 0.5(1 + \cos(k))$ . A study of the head was performed on a 1.5T scanner with spin-echo imaging ( $T_R=2000$  ms,  $T_E=10$  ms, flip angle  $90^\circ$ , slice thickness 5 mm, matrix  $256 \times 256$ , FOV  $260 \times 260$  mm<sup>2</sup>) and a 4 channel receiver coil. In k-space density weighted (DW) imaging the DVF is realized at optimal SNR by sampling k-space with a sampling density proportional to the Fourier transform of the DVF [3]. Therefore, in phase-encoding direction, both a Cartesian sampled and a non-Cartesian Nyquist sampled acquisition whose sampling density was Hanning shaped (i.e. proportional to the Fourier transform of the desired voxel function) were applied. Additionally, a noise scan was performed. The SNR was calculated with the pseudo replica method [4]. The resulting voxel functions (RVF) were calculated with:  $f_{\rho}(r) = \sum_{\rho, \kappa} F_{\rho, (\gamma, \kappa)} s_{\gamma}(r) e^{ik_{\kappa} r}$ .

**Results:** SNR maps derived using SENSE imaging with the WVA and SVA are shown in Figure 1 for both samplings. In comparison to the Cartesian acquisition, the SNR is increased for the non-Cartesian data by 8% in case of the WVA and by 17% in case of the SVA. For both samplings the SNR is higher in case of the SENSE reconstruction using the SVA. The resulting voxel functions are shown in Figure 2. Using the SVA the RVF is identical to the DVF for both samplings. In contrast, with the WVA the RVF has a similar mainlobe as the desired voxel function with increased sidelobes next to the mainlobe.

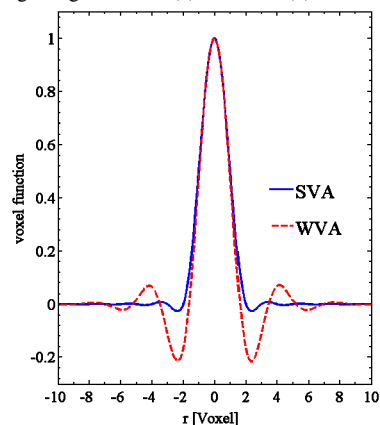
**Discussion:** In the WVA the resulting voxel function is constrained to be orthonormal to the DVF. The neighboring voxel functions resulting from the Fourier transform of the Hanning window overlap at the edges of the voxels. Therefore, mainly negative sidelobes occur in the resulting voxel function to fulfill the orthonormality to neighboring voxel functions. The SVA provides the desired voxel function as a consequence of the least squares minimization. The SNR increase of 17% achieved with the SVA for the non-Cartesian sampling compared to the Cartesian sampling is in agreement to the theory of DW imaging [3]. The reduced SNR gain of WVA for the non-Cartesian sampling in comparison to the Cartesian sampling is caused by the RVF differing from the desired voxel function. Therefore, the sampling density and the Fourier transform of the RVF function do not match perfectly. The increased SNR of SVA compared to WVA for the identical sampling can be explained by the differing resulting voxel function. The increased negative sidelobes of the voxel function of WVA result in an increased negative contamination by neighboring voxels.

**Conclusion:** The advantage of SVA compared to the commonly applied WVA is that the resulting voxel function coincides with the DVF and additionally allows to be combined with the optimal sampling strategy for the DVF known from DW imaging [3].

**References:** [1] Pruessmann KP, et al. Magn Reson Med 1999, 42: 952. [2] Gutberlet M. Proceedings ISMRM 2012, 2221. [3] Greiser A, et al. Magn Reson Med 2003, 50: 1266. [4] Robson PM, et al. Magn Reson Med 2008, 60: 895.



**Figure 1:** SNR maps of SENSE imaging for the Cartesian sampling using the WVA (a) and SVA (b) and for the non-Cartesian sampling using the WVA (c) and SVA (d).



**Figure 2:** Resulting voxel functions of the WVA and SVA for both samplings.