Sparsity-enforced Kalman filter technique for dynamic cardiac imaging

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Introduction: Dynamic MRI is a technique to capture the object in motion with high spatial and temporal resolution, which can be achieved by reducing the total amount of acquired *k*-space data. Several strategies [1] have been proposed to reconstruct the dynamic images from partially acquired *k*-space data. Recently, Kalman filter (KF) technique has been used to model the dynamic MRI, which yields a casual model for real-time imaging [2-4]. In this work, we proposed a KF-based framework by incorporating a sparsity constraint to the KF model of [4]. Simulations of cardiac dynamic MRI were conducted to demonstrate the improved performance of the new sparsity-enforced KF technique.

Theory: Dynamic MRI can be described by a discrete linear filtering model as $\mathbf{x}_k = \mathbf{x}_{k-1} + \mathbf{w}_{k-1}$ and $\mathbf{z}_k = \mathbf{F}_k\mathbf{x}_k + \mathbf{v}_k$, where the state variable $\mathbf{x}_k$ is the image column vector assuming the row vector is along the readout direction, the measurement $\mathbf{z}_k$ is the corresponding column vector after applying Fourier transform to the row vectors of acquired *k*-space data, the measurement matrix $\mathbf{F}_k$ is partial Fourier transform matrix, $\mathbf{w}_k$ and $\mathbf{v}_k$ are system and measurement noises with $\mathbf{w}_k \sim \mathrm{N}(\mathbf{0}, \mathbf{Q}_k)$ and $\mathbf{v}_k \sim \mathrm{N}(\mathbf{0}, \mathbf{R}_k)$, respectively [4]. The KF technique gives a recursive solution to the above linear filtering model using $\hat{\mathbf{x}}_k^+ = \hat{\mathbf{x}}_{k-1}^+ + \mathbf{P}_k^-\mathbf{F}_k^T[\mathbf{F}_k\mathbf{P}_k^-\mathbf{F}_k^T + \mathbf{R}_k]^{-1}[\mathbf{z}_k - \mathbf{F}_k\hat{\mathbf{x}}_{k-1}^+]$, where $\mathbf{P}_k^-$ is the estimation error covariance matrix which is updated by $\mathbf{P}_{k+1}^- = \mathbf{P}_k^- - \mathbf{P}_k^-\mathbf{F}_k^T[\mathbf{F}_k\mathbf{P}_k^-\mathbf{F}_k^T + \mathbf{R}_k]^{-1}\mathbf{F}_k\mathbf{P}_k^- + \mathbf{Q}_k$. However, this KF technique has two limitations: (a) The $\mathbf{P}_k^-$ only depends on the $\mathbf{F}_k$ and system noise covariance matrix $\mathbf{Q}_k$, and it does not depend on the estimation of the state variable; (b) This technique don't impose additional constraints, such as sparsity or total variation (TV), on the state variable. Provided that the measurement noise covariance matrix $\mathbf{R}_k$ is diagonal and time-invariant, a few lines of algebra shows that this KF technique yields the same solution as the following constrained minimization problem: $\min \mathbf{u}_k^H(\mathbf{P}_k^-)^{-1}\mathbf{u}_k, \mathrm{s.t.} ||\mathbf{z}_k - \mathbf{F}_k\hat{\mathbf{x}}_{k-1}^+ - \mathbf{F}_k\mathbf{u}_k||_2 < \epsilon$. And $\hat{\mathbf{x}}_k^+ = \hat{\mathbf{x}}_{k-1}^+ + \hat{\mathbf{u}}_k$, where $\hat{\mathbf{u}}_k$ is the difference between the frame being reconstructed and previous frame. Because this difference is usually sparse, a sparsity constraint can be added and then we can format the optimization problem as $\min [\mathbf{u}_k^H(\mathbf{P}_k^-)^{-1}\mathbf{u}_k + \alpha\mathbf{u}_k^H\mathbf{W}_k\mathbf{u}_k], \mathrm{s.t.} ||\mathbf{z}_k - \mathbf{F}_k\hat{\mathbf{x}}_{k-1}^+ - \mathbf{F}_k\mathbf{u}_k||_2 < \epsilon$, where $\mathbf{W}_k$ is a diagonal weighting matrix to enhance the sparsity of $\mathbf{u}_k$, and it is chosen iteratively by $\mathbf{W}_{k,i} = \mathrm{diag}(|\mathbf{u}_{k,i-1}|^{-1})$ [5]. The iteration terminates when $||\mathbf{u}_{k,i} - \mathbf{u}_{k,i-1}||_2/||\mathbf{u}_{k,i}||_2 < \tau$. Since $\mathbf{u}_k^H(\mathbf{P}_k^-)^{-1}\mathbf{u}_k + \alpha\mathbf{u}_k^H\mathbf{W}_k\mathbf{u}_k = \mathbf{u}_k^H[(\mathbf{P}_k^-)^{-1} + \alpha\mathbf{W}_k]\mathbf{u}_k$, this means that the estimation error covariance matrix is corrected iteratively by the new estimation of the state variable.

Method: 25 frames of full cardiac cine was acquired using a 1.5T Philips system with 345mm×270mm FOV and 256×256 resolution. The data was obtained using a steady-state free precession (SSFP) sequence with a flip angle of 50 degree and TR=3.45ms. The *k*-space was under-sampled using Gaussian random pattern along the phase-encoding direction. For casual model, it's reasonable to use a variable-density under-sampling pattern with denser sampling at the earlier of the imaging than that of later. In this work, the 1st frame was under sampled by a factor of 2, 2nd frame by 4 and the rest of frames by 8. The 1st frame was reconstructed by simply inverse Fourier transform and the rest frames were reconstructed by the proposed method. To eliminate the effect of randomness, 256 Monte Carlo under-samplings were evaluated to find the mean and standard deviation of normalized MSE (NMSE). The proposed method was compared with KF technique [4] and k-t FOCUSS [5]. For all experiments, the parameter setting is $\mathbf{W}_{k,0} = 0, \alpha = 2$ and $\tau = 0.05$.

Results and Discussion: Fig.1 (a) shows the region of interest (ROI) and a readout position (x=152) of the x-t space. Fig.1 (b) illustrates that the proposed method outperforms the KF technique. For 11-15th frames corresponding to the moments during rapid cardiac deformation, both KF technique and proposed method achieved lower NMSE than k-t FOCUSS. Fig.1 (c) compares the x-t space at the position (x=152) and its error, and the white arrows mark the obvious differences between two KF-based methods. Fig.2 shows the result in the ROI and its error of 7, 11 and 24th frames, respectively, and the white arrows mark where the proposed method outperforms the KF technique. Furthermore, k-t FOUCSS failed to reconstruct a sharp change marked by red arrows in Fig.1 (c) and Fig.2(b), which were well reconstructed by the two KF-based methods, and the proposed method was able to capture even sharp changes.

Conclusion: This work presented an improvement to the Kalman filter based dynamic MRI. In this new method, the Kalman filter is firstly casted into a framework of optimization, and then a sparsity constraint is added to the framework to better capture the motion of the object. Simulation results clearly demonstrated the strength of the sparsity-enforced Kalman filter method. Future work will seek combination of additional a prior information for the Kalman filter reconstruction of dynamic MRI with higher spatial and temporal resolution.

References: [1] Jeffrey Tsao et al, JMRI, 36, 2012. [2] Namrata Vaswani, ICIP'08, pp893-896. [3] Uygar Sümbül et al, IEEE Trans. Med. Imag., 28, 2009. [4] X Feng et al, MRM, to appear, 2012. [5] H Jung et al, PMB, 52, 2007.

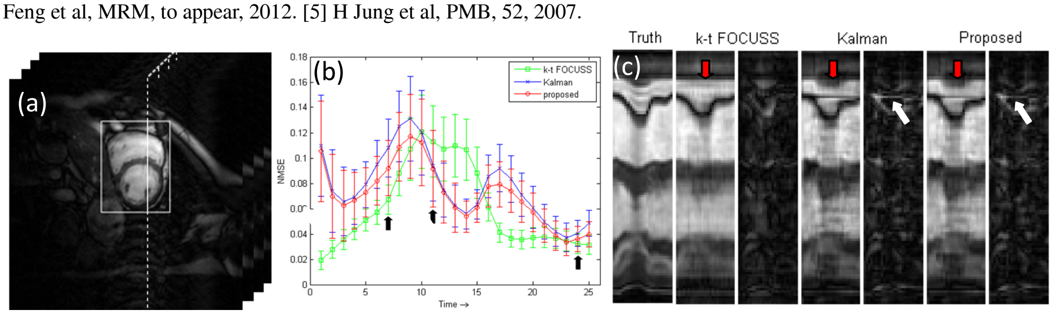

Fig. 1 (a) shows the ROI and one position (x=152) of x-t space; (b) shows the mean and standard deviation of NMSE, and the 3 black arrows indicates 3 representative frames inspected in Fig. 2; (c) shows the x-t space at x=152 within ROI indicted in (a).

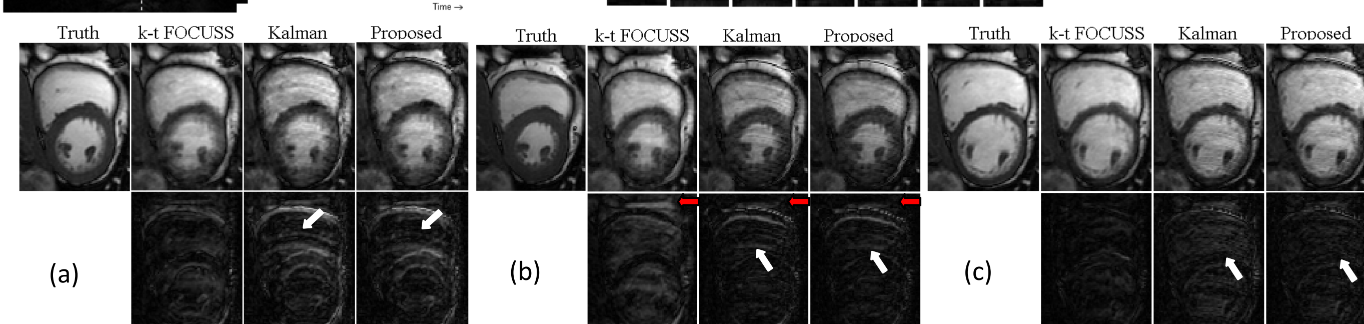

Fig. 2 shows the ROI and error of 7(a), 11(b) and 24th(c) frames marked with black arrows in Fig. 1(b).