Lower Extremities Perfusion Imaging with Low-Rank Matrix Completion Reconstruction

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Target Audience: MR engineers, physicists and clinicians interested in image reconstruction, perfusion imaging, dynamic contrast enhanced (DCE) imaging and lower extremities imaging.

Purpose: An accurate measurement of lower extremities perfusion is potentially of significant help in the assessment of peripheral arterial disease (PAD). First-pass DCE imaging is challenging in the lower extremities because both sufficient volumetric coverage and high temporal resolution (< 2 seconds) are desired. Fast non-Cartesian trajectories such as stack of spirals[1] are promising candidates but are subject to off-resonance blurring effects. For an equivalent scan time using 3DFT imaging, high scan acceleration is required. We investigate the use of the low-rank matrix completion reconstruction[2] for highly accelerated DCE perfusion imaging in the lower extremities.

Methods: Because of information redundancy in multiple time frames and the fact that the dynamics of perfusion can be described by a small number of basis functions, reshaping the image series into Casorati form yields a low-rank matrix X ($n \times t$ matrix, where n is the number of pixels per image and t is the total number of time frames)[3]. Moreover, dividing images into small blocks X_b has even better low-rank property[4]. This low-rank matrix X can be recovered from the undersampled k-space data y by solving:

$$\min_x \Sigma||X_b||_* + \mu||A(X) - y||^2$$

$||X_b||_*$ is the sum of singular values of X_b and μ is the regularization coefficient trading off $\Sigma||X_b||_*$ and the measurement error. This problem can be solved efficiently by the fixed-point continuation method[5]. Finally, the image series can be obtained by reshaping X.

In perfusion imaging, signal amplitude can be considered as static before injection or after peak-enhancement. Therefore we propose to fully sample several time frames at the beginning and the end of the scan when temporal resolution is not constrained. Fully sampled frames serve as reference images for undersampled frames and improve the reconstruction fidelity.

In each time frame, a calibration region is fully sampled and the outer portion of k-space is undersampled. The undersampling pattern is designed to be uniformly random (Bernoulli distribution) and unique for each time frame. In addition, the data at consecutive frames are complementary, i.e., neighbouring frames collect different k-space data but they can be combined into a fully sampled data set (Fig. 1). The number of time frames needed for a fully sampled dataset equals the acceleration factor in the outer portion of k-space. This pattern is preferred because the missing k-space data can be more accurately estimated if the k-space data at the same position is acquired in neighbouring time frames.

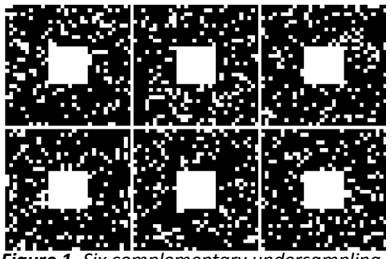

Figure 1. *Six complementary undersampling patterns, combined to be fully sampled.*

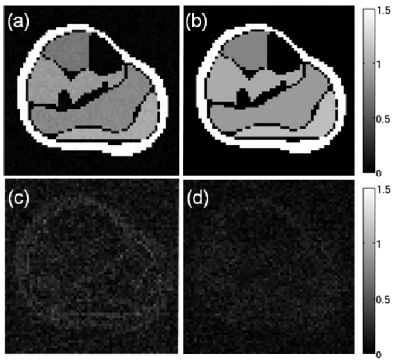

Figure 2. *Simulation. (a) fully sampled image. (b) reconstructed image with reference images, agreeing well with fully sampled image.* ***10 times*** *error map between noiseless images and reconstruction (c) without and (d) with reference images. With reference images, the error is greatly reduced.*

Results and Discussion: Numerical simulations were performed to compare the low-rank matrix reconstructions with and without reference images (Fig. 2). DCE behaviour of several different muscle groups[6] and fat were simulated. Gaussian noise was added to the signal. The reconstruction was compared to the noiseless images, which represent the true dynamic behaviour. Out of 60 time frames, the 1^{st}~4^{th} and 57^{th}~60^{th} are fully sampled as reference images. With an acceleration factor (R) of 6.5, the normalized root-mean-square-error (RMSE) of the reconstruction with reference images is 0.0180, while the RMSE increases to 0.0261 without reference images.

The proposed reconstruction method was also tested using retrospectively undersampled in-vivo DCE data in the lower extremities obtained with a stack-of-spirals trajectory[1] (Fig. 3). The maximum slope of the signal rise was calculated as a semi-quantitative perfusion index (Fig. 3(d)-(f)). With R = 6.5, the image and maximum slope of low-rank reconstruction both agree well with fully sampled data. Also, the time curves of muscle are recovered well for the numerical simulation and the in-vivo data (Fig. 4). With R = 6.5, the temporal resolution of 3DFT can be improved to 2 seconds, covering the lower extremities with field of view = $30 \times 15 \times 32$ cm^3 and spatial resolution = $2.5 \times 2.5 \times 8$ mm^3.

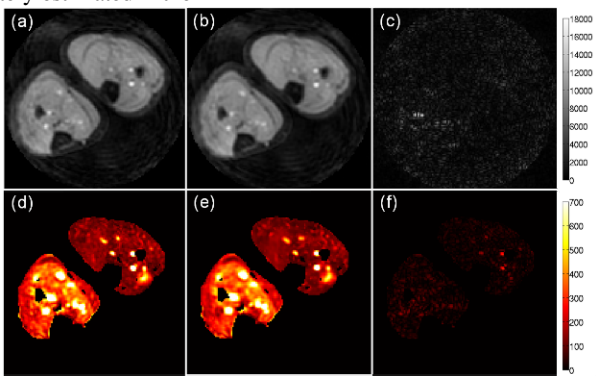

Figure 3. *In-vivo data. (a) fully sampled. (b) reconstructed image with retrospectively undersampled data. (c)* ***10 times*** *error map of image. Maximum slope map calculated based on (d) fully sampled data and (e) undersampled data. (f) error map of maximum slope. Both images and maximum slope agree well with fully sampled data.*

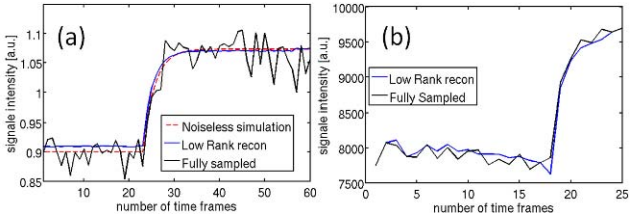

Figure 4. *Time curve of low-rank reconstruction.(a) numerical simulation. (b) retrospectively undersampled in-vivo data. The temporal behavior was recovered successfully with low-rank reconstruction.*

Conclusion: With reference images and an optimized undersampling pattern, the low-rank matrix completion method facilitates high scan acceleration for volumetric DCE perfusion imaging of the lower extremities. The proposed acquisition and reconstruction method enables accelerated 3DFT imaging with volumetric coverage of $30 \times 15 \times 32$ cm^3 and temporal resolution of 2 seconds.

References: **[1]** Lee JH, *et al.* MRM 50: 1276–1285, 2003; **[2]** Candés EJ, *et al.* CACM 55: 111-119, 2012; **[3]** Haldar J and Liang Z, Proc. ISBI 2010; **[4]** J Trzasko, *et al.* Proc. 19th ISMRM, p. 298, 2011; **[5]** Ma S, *et al.* Tech. Rep., 2008; **[6]** Thompson R, *et al.* MRM 54: 289–298, 2005;