Evaluation of Systematic and Statistical Reconstruction Errors in Compressed Sensing Reconstructions

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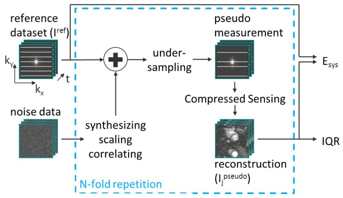
Introduction

In Compressed Sensing (CS) (1–3) comprehensive metrics for evaluating the image quality are hardly available. Therefore, in this work, a simple Monte Carlo approach is introduced that allows estimating the systematic and statistical reconstruction errors for CS reconstruction frameworks.

Theory

In CS images are reconstructed from undersampled data by exploiting data redundancy in terms of image or transform sparsity. The sparsity thereby acts as a kind of signal model to the reconstruction. Corresponding to an inaccurate signal model, a lack of sparsity will result in systematic errors. Similarly, a bad choice of the undersampling pattern will result in non-removable undersampling artifacts, i.e reconstruction errors of systematic nature. A simple way to retrieve systematic errors would be to perform a large number of independent measurements and to examine the deviations of their reconstructions from a fully sampled reference. This of course is impractical. A more suitable way is to synthesize the measurements utilizing a Monte Carlo simulation similar to (4). To generate the pseudo measurements, a fully sampled reference dataset is retrospectively undersampled after adding artificially generated, correctly scaled and coil correlated noise (Fig. 1). Each pseudo measurement is reconstructed. From the results, the systematic error pixelwise can be calculated as given by equation [1]. Thereby I_j^{pseudo} represents the absolute value of the reconstruction of the j^{th} pseudo measurement, I^{ref} the corresponding complete reference and *N* the number of pseudo measurements. Information on statistical signal deviations is also provided. As the non-linearity of CS impairs quantitative noise estimations, descriptive statistics may be utilized. A robust measure of data fluctuation is the quartile metric. The interquartile range is defined as the signal intensity range between the first (lower) quartile (*Q1*) and third (upper) quartile (*Q3*) of all signal intensities I_j^{pseudo} (see equation [2]). The lower and upper quartile thereby cut off the lowest and highest 25 % of the data values.



$$E_{sys} = \left({}^{1}/_{N} \textstyle\sum_{j=1}^{N} I_j^{pseudo}\right) - I^{ref} \qquad [1]$$

$$IQR(x,y) = Q_3(x,y) - Q_1(x,y) \qquad [2]$$

***Fig. 1:** Schematic view of the Monte Carlo simulation for error evaluation. The equations for calculating the systematic error (E_{sys}) and the interquartile range (IQR) are given.*

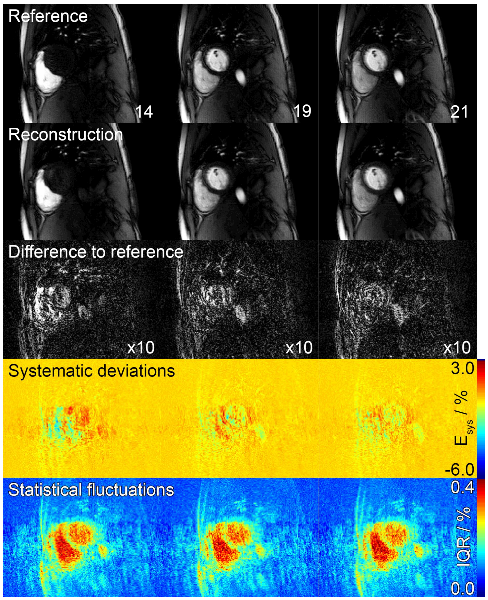


***Fig. 2:** Results of the error evaluation. The systematic error (E_{sys}) and interquartile range (IQR) are given in % relative to the maximum signal intensity of the perfusion series.*

Material and Methods

The proposed approach was utilized to evaluate the systematic and statistical errors of a CS reconstruction in contrast enhanced myocardial first-pass perfusion MRI. Measurements were performed on a clinical 3T Magnetom Trio system (Siemens Healthcare Sector, Erlangen, Germany) equipped with a 32 channel cardiac array (Siemens Healthcare Sector, Erlangen, Germany). A fully sampled reference dataset was obtained from one volunteer utilizing a saturation prepared FLASH sequence (FOV: 320 x 360 mm^2; matrix: 160 x 180; slice thickness: 8 mm; TI: 175 ms; TE: 1.44 ms; TR: 2.5 ms; T_{Acq}: 323 ms; GRAPPA: 1.5-fold). Each time frame was undersampled retrospectively by a factor of 5 using a different sampling pattern with the phase encoding lines homogeneously distributed in k-space. The central 9 lines were included in every pattern. For image reconstruction, an Iterative Soft Thresholding algorithm (5) was employed using a Fourier transform along the time dimension (x-f-space) as sparsifying transform. The error evaluation was performed as described above.

Results

The results of the image reconstruction and error evaluation are displayed in Fig. 2. Shown are the reference (top row), the reconstruction (2nd row), the difference between the both (3rd row), the systematic signal deviations (4th row) as well as the IQR (5th row), each for representative time frames (indicated in 1st row). Visually, the image reconstruction performs well and no significant differences can be seen between reconstruction and reference. The error evaluation reveals systematic deviations that exceed the statistical ones by far. Thus, they correspond to the intensity difference between the reference and the reconstruction. While the systematic errors vary with the frame, the statistical fluctuations seem to be similar for all frames. During right ventricle (RV) peak enhancement, a systematic signal underestimation can be seen within the RV (frame 14). Similarly, the signal in the left ventricle (LV) is underestimated during the LV peak enhancement (frame 19). Within the same frame, the myocardial signal is overestimated. During the course of contrast uptake, this overestimation reduces (frame 21) and turns into underestimation (not shown). The up-slope of the contrast enhancement consequently is significantly underestimated.

Discussion

CS reconstructions entail systematic errors that may have severe impact onto the image quality. In the presented case, systematic errors significantly bias the contrast uptake that is visible in the ventricles and the myocardium and may falsify the results of perfusion quantification. The errors are the result of the suppression of higher temporal frequencies in x-f-space during the reconstruction. It leads to severe blurring in the temporal dimension and a flattening of the intensity time courses. The proposed approach for error evaluation helps identifying sources of systematic errors in the CS concept that is utilized for image reconstruction.

References

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