Pseudo-random Center Placement O-space Imaging: Optimizing Incoherence for Compressed Sensing

Leo K Tam[1], Gigi Galiana[2], Jason P Stockmann[3], Andrew Dewdney[4], Terence W Nixon[2], Dana C Peters[2], and R Todd Constable[2,5]

[1]Biomedical Engineering, Yale University, New Haven, CT, United States, [2]Diagnostic Radiology, Yale University, New Haven, CT, United States, [3]Martinos Center, Massachusetts General Hospital, Boston, Massachusetts, United States, [4]Siemens AG Healthcare, Erlangen, Bavaria, Germany, [5]Neurosurgery, Yale University, New Haven, CT, United States

Target Audience: The parallel imaging, compressed sensing, and non-linear gradient encoding communities. **Purpose**: O-space imaging utilizes the Z^2 (in-plane X^2+Y^2) non-linear gradient for spatial encoding tailored to complement the parallel receiver array spatial information and maximize data efficiency for high acceleration factors.[1] Another acceleration technique, compressed sensing (CS), uses the sparsity of MR images in the wavelet domain by sampling k-space in a pseudo-random (PR) manner and applying a sparsity promoting reconstruction algorithm.[2] CS requires that the acquisition produce incoherent artifacts in the sparse domain. We created an O-space sequence with PR center placements (CPs) to further distribute artifacts in the sparse domain. The current work proposes a PR CP strategy for O-space imaging with optimization of incoherence within the CS framework and compares it to traditional PR linear gradient k-space acquisitions.

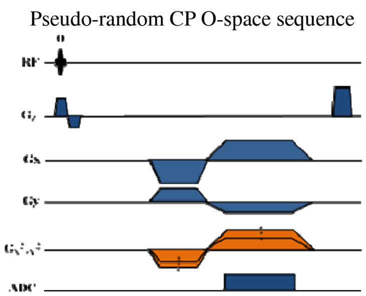

Fig. 1. The PR CPs O-space imaging sequence pseudo-randomizes the Z^2 strength within 30% of the nominal strength based on incoherence.

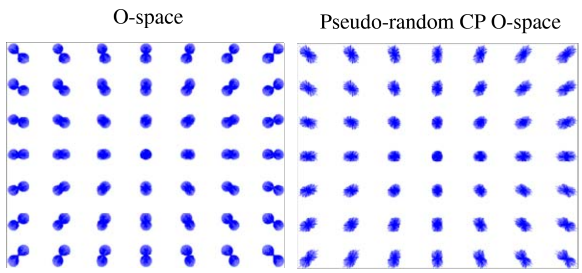

Fig. 3. Local k-space for a 7x7 grid of representative points. From the local k-space, we infer the reconstruction at a given point will behave similarly to that k-space. O-space does not maximize incoherence. Pseudo-random CP O-space radius has two effects, 1) increasing incoherence and 2) improving the k-space coverage.

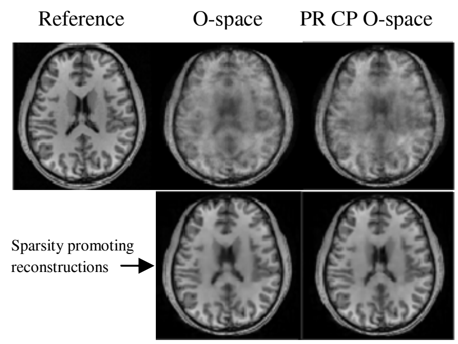

Fig. 4. Simulation results show PR CPs O-space at R=32, 256x256 and Ns=512. With the sparsity promoting reconstruction, the PR CPs improved the image (SSE = 32.06 vs 41.31).

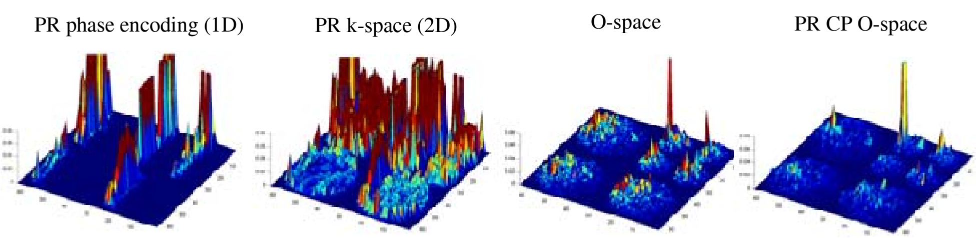

Fig. 2. Sample TPSFs for several acquisition methods are shown for an arbitrary point at coordinates (7, 29) in the wavelet domain. The TPSF with highest peak and low noise-like background denotes greatest incoherence and suitability for the CS approach. The spatially-varying nature of the acquisition is examined with local k-space figures (see fig. 3). The pseudo-random O-space imaging shows interference patterns that spread more evenly across length scales and orientations.

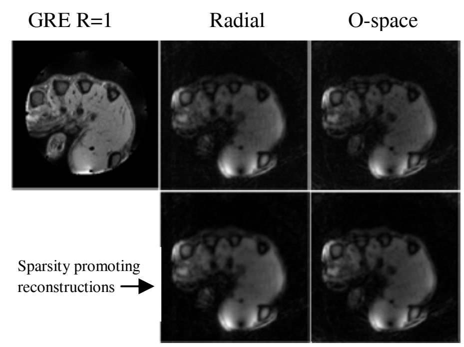

Fig. 5. Preliminary *in vivo* hand reconstructions at R=8 256x256. One feature that comes through better in the sparsity enforcing reconstructions is the isolated finger.

Methods: The pulse sequence (fig. 1) shows the PR CPs O-space is a gradient echo projection imaging sequence (no phase encoding) with the Z^2 gradient amplitude for each CP pseudo-randomized by up to 30%. The Z^2 coefficients are iterated 20 times to find a best "randomized" set to produce maximum incoherence. Experiments were performed using a Z^2 gradient insert from Resonance Research, Inc. (Billerica, MA, USA) placed in a 3T Trio (Siemens AG, Erlangen, Germany) with a custom-built eight-channel transmit-receive array. The acquisition may be represented algebraically as a system of linear equations, y = Ax, where A is known as the encoding matrix in parallel imaging or the sensing matrix in CS. Whereas a parallel CS approach may use an iterative SENSE with non-uniform Fast Fourier Transform (NUFFT) convex sparsity-promoting reconstruction[3,4], the approach here replaces the SENSE and NUFFT methods with the Kaczmarz iterative algebraic projection reconstruction (ART) algorithm.[3] Converging to the minimum norm solution, the ART method iterates through data provided by each coil. The sparsity-promoting convex optimization has the objective function: $\mathrm{Obj}(x) = \|A W' x - y\|^2 + \lambda_1|x|_1 + \lambda_2 TV(W'x)$, where W represents the Daubechies wavelet transform.[2] The transform point spread function (TPSF) describes incoherence by relating how one coefficient in the transform domain (sparse domain) is affected by the acquisition (see fig. 2). Mathematically, $TPSF(i,j) = e_j' WA' AW' e_i$, where e represents a point in the transform domain.[2] With maximal incoherence, the undersampling performed during the acquisition generates a low noise-like background in the sparse domain.

Results: The PR CPs O-space (fig. 2) shows better transform incoherence and local k-space coverage than O-space or PR k-space while retaining the straight-forward acquisition of gradient echo projection imaging. Simulations comparing O-space and PR CPs O-space (fig.4) show that the sparsity-promoting reconstruction reduced the sum-of-squared error (SSE) by 22.39%. Experimental evidence (fig. 5) shows O-space (no PR) corroborates initial predictions.

Discussion: Compressed sensing and pseudo-randomized O-space imaging are complementary methods that allow further accelerations in parallel imaging. O-space imaging has efficiency gains in part through the use of an imaging gradient that is complementary with receiver coil sensitivity profiles. CS, which relies on sparsity, collects data efficiently to iteratively dissipate aliasing artifacts in the sparse domain. With non-linear gradients, the necessary incoherence for CS may be optimized through methods not available to linear gradients.

References: [1]Stockmann, J. et. al. MRM 2010. 64: p. 447-456. [2]Lustig M., et al. MRM 2007. 58: p. 1182-1195. [3]Tam L.K.., et al., ISMRM, 2011, p.2896. [4]Liang et al. MRM 2009. 62: p. 1574-1584. [4]King KF ISMRM 2008:1488. [5]Herman G.T. et. al. J. Theor. Biol. 42:1.

Acknowledgements: Scott McIntyre and Peter Brown at Yale. This work was supported by NIH BRP R01 EB012289-01 and a NSF Graduate Research Fellowship.