An Efficient Compressed Sensing Reconstruction Robust to Phase Variation on MR Images

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Introduction: The application of compressed sensing (CS) to MRI has the potential to significantly reduce scan time. However, the quality of reconstructed images will be degraded when the MR images have phase variations. In the present paper, we present a new CS method that is robust to phase variations in MR images. When the signal trajectory in k-space is symmetrical with respect to its origin, the *k*-space signal corresponding to the real and imaginary parts of the complex image can be synthesized independently by restricting the k-space signal to an even function or an odd function. The proposed method involves random but symmetrical *k*-space acquisition and independent reconstruction of the real and imaginary parts of images using the real-valued constraint.

Theory: Let the observed MR signal and spin density distribution be s(**k**) and $\rho(\mathbf{x})$, respectively, where **k** is a *k*-space vector and **x** is a space vector. Then, we have

$$s(\mathbf{k}) = \int \rho(\mathbf{x}) e^{-j\phi(\mathbf{x})} e^{-j(\mathbf{k}\cdot\mathbf{x})} d\mathbf{x} = \mathrm{F}\left[\int \rho(\mathbf{x}) e^{-j\phi(\mathbf{x})}\right] \quad \cdots (1)$$

where $\phi(\mathbf{x})$ is the function of phase variation due to imperfection in the MRI equipment and inhomogeneities in the main static magnetic field, and F is the operator of the Fourier transform. Image reconstruction can be performed by applying the inverse Fourier transform to the signal s(**k**), $\rho(\mathbf{x})\exp\{-j\phi(\mathbf{x})\}=\mathrm{F}^{-1}[s(\mathbf{k})]$. The real and imaginary parts of the complex image $\rho(\mathbf{x})\exp\{-j\phi(\mathbf{x})\}$ can be written as follows:

$$\mathrm{F}\left[\mathrm{Re}\left[\rho(\mathbf{x})e^{-j\phi(\mathbf{x})}\right]\right] = \frac{1}{2}\mathrm{F}\left[\rho(\mathbf{x})e^{-j\phi(\mathbf{x})} + \rho(\mathbf{x})e^{j\phi(\mathbf{x})}\right] = \frac{1}{2}\left\{s(\mathbf{k}) + s(-\mathbf{k})^*\right\} \quad \cdots (2),$$

$$\mathrm{F}\left[\mathrm{Im}\left[\rho(\mathbf{x})e^{-j\phi(\mathbf{x})}\right]\right] = -\frac{j}{2}\mathrm{F}\left[\rho(\mathbf{x})e^{-j\phi(\mathbf{x})} - \rho(\mathbf{x})e^{j\phi(\mathbf{x})}\right] = -\frac{j}{2}\left\{s(\mathbf{k}) - s(-\mathbf{k})^*\right\} \quad \cdots (3)$$

If a symmetrical relation with respect to the origin of *k*-space is provided, as shown in Eq. (2) and (3), then the real and imaginary parts of complex images can be reconstructed independently of each other. In the present paper, we focus on Cartesian grid sampling, which is the most widely used technique. In general CS, random point sampling is adopted for the phase encoding direction, because scan time reduction is exactly proportional to the undersampling factor. In the proposed method, the sampling trajectory for the phase encoding direction is set so as to be random but is symmetrical with respect to the origin of *k*-space, which makes the calculation of Eq. (2) and (3) possible using the acquired under-sampled signal data. Let $s_{cs}(\mathbf{k})$ be the acquired signal data. Then, theoretically, CS reconstruction using $s_{csr}(\mathbf{k})=1/2\{s_{cs}(\mathbf{k})+ s_{cs}(-\mathbf{k})\}$ and $s_{csi}(\mathbf{k})=j/2\{s_{cs}(\mathbf{k})- s_{cs}(-\mathbf{k})\}$ give the real-part and imaginary-part of the complex image $\rho_r(\mathbf{x})$, $\rho_i(\mathbf{x})$, respectively. It should be noted that we can use the rather strong constraint that the images obtained in this method are real-valued functions in each iteration procedure, which can improves the quality of the resultant image. Finally, we can obtain complex images by combining real and imaginary parts of images as $\rho(\mathbf{x})= \rho_r(\mathbf{x})+j\,\rho_i(\mathbf{x})$.

Results and Discussion: MR normal volunteer images were collected using a Toshiba 1.5T MRI scanner. Flow-sensitive black blood images were acquired in order to obtain images that have locally varying phase distortions due to blood flow, as well as gently varying distortions due to static field inhomogeneities (TE/TR = 40/50 ms, 256×256 matrix, slice thickness: 1.5 mm × 50 slices). The signal for the phase encoding direction, except for the central region, is randomly selected in order to simulate a given reduction factor, as shown in Fig. 1(a). Figure 1(b) shows the proposed CS trajectory in which echo signals were randomly selected but is symmetrical with respect to its origin. Reconstruction was performed using an iterative soft thresholding algorithm[2]. We used the FREBAS transform[3] as a sparsifying function, which is a kind of directional image decomposition algorithm. Figure 2 shows the peak-signal-to-noise ratio (PSNR) for two representative images, A and B, shown in Fig. 3, as a parameter of signal reduction factor. The PSNR was shown to be dramatically improved in the proposed method by using the real-valued constraint for the obtained images. We compare four reconstruction methods, as shown in the table of Fig. 3. Since a complex sparsifying transform is not required in the proposed method, we attempted to reconstruct images using a real-valued wavelet transform (SymCS-W). Since the FREBAS transform is a complex-valued transform and complex images can be directly transformed to sparsified space, we attempted to reconstruct images without the use of the real-valued constraint or phase correction (CS-cmplx). For comparison, CS reconstruction using the real-valued constraint after correcting the phase variation using the estimated phase function that was provided by the central region of the *k*-space signal was performed (CS-PC). Figures 3 indicates that the real-valued wavelet transform is also applicable to the proposed method. The best performances were obtained using the real-valued constraint and the FREBAS transform.

Symmetrical sampling reduces the mutual incoherence between the sampling operator and the Fourier transform operator, which may degrade the resultant images. However, the effect of the real-valued constraint on removing the error components of the images is greater because the imaginary parts of the temporally reconstructed images are removed. Therefore, the resultant images have improved PSNR and smaller artifacts.

Conclusion: A new CS technique that is robust to phase variations is proposed. The proposed method reconstructs the real and imaginary parts of images independently using the real-valued constraint. Several numerical experiments demonstrated that the proposed CS method provides better-quality images compared to simple reconstruction using a complex-valued sparsifying transform function or reconstruction with phase correction by applying estimated phase functions to the images.

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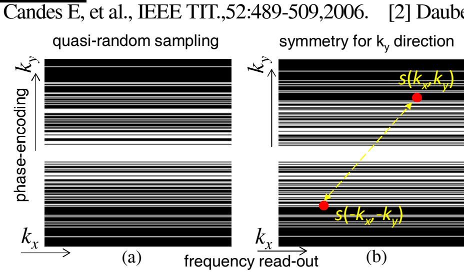

Fig. 1 Signal trajectory: (a) standard, (b) proposed.

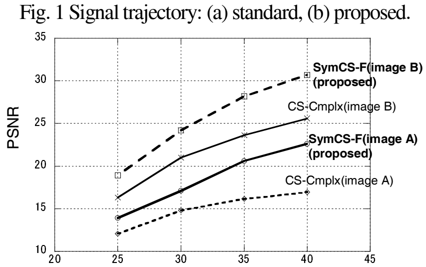

Fig. 2 PSNRs of the obtained images.

	fully scanned	SymCS-F (proposed)	SymCS-W	CS-cmplx	CS-PC
Phase variation	Yes	Yes	Yes	Yes	Yes
sparsifying func.	-	FREBAS	wavelet	FREBAS	FREBAS
signal trajectory	fully scanned	symmetrical	symmetrical	random	random
Real-value constraint	-	Yes	Yes	No	Yes
Phase correction	-	No	No	No	Yes

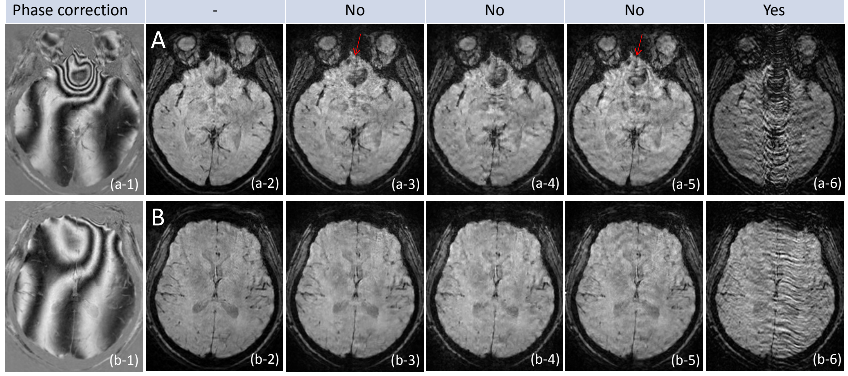

Fig. 3 Comparison of reconstructed images with other reconstruction methods.